Space Charge Transport Limits of Ion Beams in Periodic Quadrupole Focusing Channels

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Outline

- Background and Review of Stability Results
- Particle-in-Cell Simulations of Beam Stability
- Can KV-like Modes Explain Observed Instabilities?
- Core-Particle Model
- Core-Particle Simulations
- Conclusions
Good transport of a single component beam with intense space-charge described by a Vlasov-Poisson type model requires:

**Lowest Order:**
1. Stable single-particle centroid

**Next Order:**
2. Stable rms envelope

**Higher Order:**
3. “Stable” Vlasov description

Transport can fail or become “unstable” within the Vlasov model for several reasons:

- Collective modes internal to beam become unstable and grow
  - Large amplitudes can lead to statistical (rms) beam emittance growth
- Excessive halo generated
  - Increased statistical beam emittance and particle losses
- Combined processes above
Beam stability in periodic focusing lattices will be analyzed -- assume piecewise constant applied focusing forces.

- **Lattice Period** $L_p$
- **Occupancy** $\eta$ 
  $\eta \in [0, 1]$
- **Solenoid description** carried out implicitly in Larmor frame [see Lund and Bukh, PRST­ Accel. and Beams 7, 024801 (2004)]
- **Syncopation Factor** $\alpha$
  $\alpha \in [0, \frac{1}{2}]$
  
  \[ \alpha = \frac{1}{2} \implies FODO \]
Undepressed particle phase advance $\sigma_0$ measures the strength of the applied focusing functions $\kappa_x(s)$ of periodic lattices.

Single-particle orbit without space-charge:

$$x'' + \kappa_x(s)x = 0$$

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = M_x(s \mid s_i) \cdot \begin{pmatrix} x(s_i) \\ x'(s_i) \end{pmatrix} \quad M_x = \text{2 x 2 Transfer Matrix from} \quad s = s_i \text{ to } s.$$ 

Undepressed particle phase advance:

$$\cos \sigma_0 = \frac{1}{2} \text{Tr} M_x(s_i + L_p \mid s_i)$$
Lowest level description transport requires a stable centroid/single-particle orbit

Centroid:

\[ X = \langle x \rangle_\perp \]
\[ Y = \langle y \rangle_\perp \]

Centroid equation of motion neglecting image charges:

\[ X'' + \kappa_x(s)X = 0 \]

Stability requires:

\[ \frac{1}{2} \left| \text{Tr} M_x(s_i + L_p \mid s_i) \right| < 1 \implies \sigma_0 < 180^\circ \]

[Courant and Snyder, Annals of Physics 3, 1 (1958)]
The rms envelope equations are used to describe the beam at the next level of complexity.

**rms/KV envelope Equations:**

\[
\begin{align*}
\frac{r''_x}{r_x + r_y} + \kappa_x r_x - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_x^2}{r_x^3} &= 0 \\
\frac{r''_y}{r_x + r_y} + \kappa_y r_y - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_y^2}{r_y^3} &= 0 \\
\end{align*}
\]

\[
Q = \frac{q\lambda}{2\pi\epsilon_0 m\gamma_b^3 \beta_b^2 c^2} = \text{const} \quad \text{perveance}
\]

\[
\varepsilon_x = 4[\langle x^2 \rangle_\perp \langle x'^2 \rangle_\perp - \langle xx' \rangle_\perp^2]^{1/2} \quad \text{rms edge emittance}
\]

\[
\varepsilon_x = \text{const}, \quad \varepsilon_y = \text{const} \quad \text{for KV model exact, approx true most other cases}
\]
A matched beam envelope provides the most efficient (radially compact) transport in a periodic focusing channel.

Periodic “matched” envelope conditions:

\[
\begin{align*}
 r_x(s + L_p) &= r_x(s) \\
 r_y(s + L_p) &= r_y(s)
\end{align*}
\]

**Example Parameters**

\[
\begin{align*}
 L_p &= 0.5 \text{ m}, \quad \sigma_0 = 80^\circ, \quad \eta = 0.5 \\
 \varepsilon_x &= \varepsilon_y = 50 \text{ mm-mrad} \\
 \sigma/\sigma_0 &= 0.2
\end{align*}
\]

**Solenoidal Focusing**

\(Q = 6.6986 \times 10^{-4}\)

**FODO Quadrupole Focusing**

\(Q = 6.5614 \times 10^{-4}\)
The depressed particle phase advance provides a convenient measure of space-charge strength

Depressed single-particle phase advance in the presence of uniform space-charge for a particle moving in the matched beam envelope:

$$\varepsilon_x = \varepsilon_y \equiv \varepsilon$$

$$\sigma = \varepsilon \int_{s_i}^{s_i + L_p} \frac{ds}{r^2_x} = \varepsilon \int_{s_i}^{s_i + L_p} \frac{ds}{r^2_y}$$

$$\lim_{Q \to 0} \sigma = \sigma_0$$

Normalized space charge strength or “depressed tune”:

$$0 \leq \sigma / \sigma_0 \leq 1$$

$$\sigma / \sigma_0 \to 0$$  \hspace{1cm} \text{Cold Beam (space-charge dominated)}

$$\varepsilon \to 0$$

$$\sigma / \sigma_0 \to 1$$  \hspace{1cm} \text{Warm Beam (kinetic dominated)}

$$Q \to 0$$
Instability bands of the envelope equation are well understood in periodic focusing channels

Envelope Mode Instability Growth Rates

Solenoid ($\eta = 0.25$)  

Quadrupole FODO ($\eta = 0.70$)

High level transport simulations and theory are based on the “stability” of the initial distribution using a Vlasov-Poisson model.

**Vlasov equation:**

\[
\left\{ \frac{\partial}{\partial s} + \frac{\partial H_\perp}{\partial \mathbf{x}'\perp} \cdot \frac{\partial}{\partial \mathbf{x}_\perp} - \frac{\partial H_\perp}{\partial \mathbf{x}_\perp} \cdot \frac{\partial}{\partial \mathbf{x}'\perp} \right\} f_\perp(\mathbf{x}_\perp, \mathbf{x}'\perp, s) = 0
\]

**Hamiltonian:**

\[
H_\perp(\mathbf{x}_\perp, \mathbf{x}'\perp) = \frac{1}{2} \mathbf{x}'_\perp^2 + \frac{1}{2} \kappa_x x^2 + \frac{1}{2} \kappa_y y^2 + \frac{q}{m\gamma_b^3 \beta_b^2 c^2} \phi
\]

**Poisson equation:**

\[
\nabla^2_\perp \phi = -\frac{\rho}{\epsilon_0} = -\frac{q}{\epsilon_0} \int d^2 x'_\perp f_\perp
\]

+ boundary conditions on \( \phi \)

Vlasov-Poisson system is specified as an initial value problem advanced in \( s \)

- Initial distribution need not be an equilibrium
- PIC simulations can be used to model the Vlasov-Poisson evolution
Transport limits in periodic (FODO) quadrupole lattices that result from higher order processes have been measured in the SBTE experiment. These results have only a limited theoretical understanding in 20+ years.

![Experimental limits on beam stability in terms of \( \sigma \) and \( \sigma_0 \)](image)

**Low Space-Charge Intensity Transport**

**Emittance Blow Up (Unexplained)**

-- Not Practical for Applications

**Empirical fit to higher-order stability boundary**

\[
\sigma_0^2 - \sigma^2 = \frac{1}{2}(120^\circ)^2
\]

**Envelope Instability**

– Not Practical for Applications

**High Space-Charge Intensity Transport**

- Valid for Practical Applications

[M.G. Tiefenback, Ph.D Thesis, UC Berkeley (1986)]
Summary of beam stability with intense space-charge in a quadrupole transport lattice: centroid, envelope, and experiment on higher order emittance growth/particle losses
Why are we in this situation of not having a first-principles understanding of a fundamental limit of intense beam transport?

We like to think of stability in terms of an equilibrium + perturbation analysis paralleling plasma physics

- Requires an equilibrium to perform the conventional analysis

For periodic focusing, no equilibrium distributions other than uniform density KV is known:

\[ f_\perp(x, y, x', y') = \frac{\lambda}{q^2 \varepsilon_x \varepsilon_y} \delta \left[ \left( \frac{x}{r_x} \right)^2 + \left( \frac{r_x x' - r_x' x}{\varepsilon_x} \right)^2 + \left( \frac{y}{r_y} \right)^2 + \left( \frac{r_y y' - r_y' y}{\varepsilon_y} \right)^2 - 1 \right] \]

- Many instabilities from inverted population
- Low-order (envelope) structure physical
  - rms equivalent beam to model space-charge dominated beam
- Higher order “internal” modes seem to have both physical and unphysical features

For continuous focusing, many stable equilibrium solutions exist:

\[ f_\perp = f(H_\perp) \quad \text{with} \quad f_\perp(H_\perp) \geq 0 \quad \text{and} \quad df_\perp(H_\perp)/dH_\perp \leq 0 \quad \implies \quad \text{Stability} \]

- Can provide rough guide to features of periodic focused beams
- Stability may be strongly modified from continuous focusing for high \( \sigma_0 \)
WARPxy PIC simulations of quadrupole transport agree with experimental observations and show that large rms emittance growth can occur rapidly.

Parameters: $\sigma_0 = 110^\circ$, $\sigma/\sigma_0 = 0.2$ ($L_p = 0.5$ m, $\eta = 0.5$) initial semi-Gaussian

Higher $\sigma_0 > \sim 85^\circ$ makes the onset of emittance growth larger and more rapid.
Simulations suggest that transport limits observed are relatively insensitive to the structure of the initial distribution.

Parameters: $\sigma_0 = 110^\circ$, $\sigma/\sigma_0 = 0.2$ ($L_p = 0.5$ m, $\eta = 0.5$)

- Wide class of initial distributions probed – little difference in x-y plane averages but details can change in each plane due to differing wave launching conditions
- Growth becomes larger and faster with increasing $\sigma_0$
Large phase space distortions develop as the statistical emittance grows

Initial Semi-Gaussian Distribution

2 Lattice Periods

5 Lattice Periods
Distortions are similar for a wide range of smooth initial distributions.
An essential feature is that particles evolve outside the core of the beam

Take an instantaneous, rms equivalent measure of the core of the beam and “tag” particles that evolve outside the core:

\[ r_x = 2\langle x^2 \rangle^{1/2}_\perp \]

\[ r_y = 2\langle y^2 \rangle^{1/2}_\perp \]
Extensive simulations were carried out to better understand the parametric nature of the emittance growth

- All simulations carried out 6 undepressed betatron periods
  - Enough to resolve transition boundary, growth larger where weak if run longer
- Strong growth regions of initial distributions all similar (threshold can vary)
  - Irregular grid contouring with ~200 points (dots) to thoroughly probe instabilities

**initial semi-Gaussian**
- Initial thermal equilibrium almost identical

**initial Waterbag**
- Initial KV similar but does not excite envelope band
Can KV-like internal modes describe strong emittance growth? Kinetic theory would suggest instability where none is observed.

**Example: FODO Quadrupole Stability**

4\textsuperscript{th} order even mode

[Hofmann et. al, Particle Accel. 13, 145 (1983)]

σ/σ₀ → 1 (undepressed)  --->  increasing space-charge  --->  σ/σ₀ → 0 (fully depressed)

No emittance growth observed in simulations
Strong KV instabilities exist even in the continuous limit for

\[ n = 1, \text{ envelope mode curves overlap} \]

\[ \text{Red: Fluid Theory} \]
\[ \text{(no instability)} \]
\[ \text{Black: Kinetic Theory} \]
\[ \text{(unstable branches)} \]

Hoffman notation:
\[ 2n = \text{Mode “order”} \]

Notation Change:
\[ k/k_\beta \equiv \omega/\nu_0 \]
\[ \sigma/\sigma_0 \equiv \nu/\nu_0 \]

Original kinetic:

Kinetic + fluid:
KV mode eigenfunctions are internal to the core "equilibrium" beam

Continuous focusing limit eigenfunctions:

\[
\delta \phi_n = \begin{cases} 
    \frac{A_n}{2} \left[ P_{n-1} \left( 1 - 2 \frac{r^2}{r_b^2} \right) + P_n \left( 1 - 2 \frac{r^2}{r_b^2} \right) \right], & 0 \leq r \leq r_b \\
    0, & r_b < r
\end{cases}
\]

\[A_n = \text{const} \quad P_n(x) = \text{n}^{\text{th}} \text{ order Legendre polynomial}\]

Potential

\[\delta n_n = \epsilon_0 \nabla^2 \delta \phi_n / q\]

Density

- Cannot describe particles well outside core
- Also, carries little free-energy convertible to emittance
  - Unlikely to produce large growths even for large amplitudes

Can a smooth equilibrium exist for periodic focusing?

The continuous focusing equilibrium distributions suggest that Debye screening should strongly modifies the beam edge as a function of core “temperature”

\[ f_{\perp} = \frac{m \gamma_b \beta_b^2 c^2 \hat{n}}{2\pi T} \exp \left( -\frac{m \gamma_b \beta_b^2 c^2 H_{\perp}}{T} \right) \]

Continuous Focusing Thermal Equilibrium Beam

Self Consistent Beam Edge
Large envelope flutter associated with strong focusing can result in a rapid high-order oscillating force imbalance acting on edge particles of the beam

**Temperature Flutter**

Elliptical rms Equivalent Beam

\[ \varepsilon_x^2 \propto T_x r_x^2 \simeq \text{const} \quad \Rightarrow \quad T_x \propto \frac{1}{r_x^2} \]

**Example Systems**

<table>
<thead>
<tr>
<th>AG Trans: ( \sigma_0 = 60^\circ )</th>
<th>( \frac{r_{\text{max}}}{r_{\text{min}}} )(^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG Trans: ( \sigma_0 = 100^\circ )</td>
<td>( \sim 4.9 )</td>
</tr>
</tbody>
</table>

| Matching Section | \( \sim 15 \) Possible |

**Characteristic Plasma Frequency of Collective Effects**

Continuous Focusing Estimate

\[ \sigma_{\text{plasma}} \sim \frac{L_p}{r_b} \sqrt{2Q} \]

**Typical:** \( \sigma_{\text{plasma}} \sim 105^\circ /\text{period} \)

- Temperature asymmetry in beam will rapidly fluctuate with lattice periodicity
  - Converging plane  \( \Rightarrow \) Warmer
  - Diverging plane  \( \Rightarrow \) Colder

- Collective plasma wave response slower than lattice frequency
  - Beam edge will not be able to adapt rapidly enough
  - Collective waves will be launched from lack of local force balance near the edge
Flutter scaling of the matched beam envelope varies for quadrupole and solenoidal focusing

\[
\frac{r_x \mid_{\text{max}}}{\bar{r}_x} - 1 \simeq \begin{cases} 
(1 - \cos \sigma_0)^{(1-\eta)(1-\eta/2)} & \text{Solenoidal Focusing} \\
(1 - \cos \sigma_0)^{1/2} \frac{(1-\eta/2)}{2^{3/2}(1-2\eta/3)^{1/2}} & \text{Quadrupole Focusing}
\end{cases}
\]

- **Solenoids:**
  - Varies significant with both \( \sigma_0 \) and \( \eta \)
- **Quadrupoles:**
  - Phase advance \( \sigma_0 \) variation significant
  - Occupancy \( \eta \) variation weak

Based on: E.P. Lee, Phys. Plasmas, 9 4301 (2002) for limit \( \sigma/\sigma_0 \to 0 \)
From these considerations that if smooth “equilibrium” beam distributions exist for periodic focusing, then they are highly nontrivial – none have been constructed in 45+ years

Would a nonexistence of an equilibrium distribution be a problem:

- Real beams are born off a source that can be simulated
  - Propagation length can be relatively small in linacs
- Transverse confinement can exist without an equilibrium
  - Particles turn at large enough radii forming an edge
  - Edge oscillate from lattice period to lattice period without pumping to large excursions

→ Does not preclude long propagation with preserved emittance

Several involved perturbative constructions of approximate Vlasov equilibria exist that may merit more evaluation:

  - Procedure accurate/inaccurate in simulations for solenoidal/quadrupole focusing
  - Solenoidal focusing better respects continuous limit

R. Davidson, H. Qin, and P. Channell, Phys. Rev. Special Topics – Accelerators and Beams 2, 074401 (1999); 3, 029901 (2001) + others and book chapter
  - Procedure unchecked in simulation

Any equilibrium + perturbation theory is unlikely to address the essential aspect of particles evolving outside the beam envelope – try another method!
Core-Particle Model  --- Transverse particle equations of motion of a test particle

\[ x'' + \kappa_x x = -\frac{q}{m \gamma_b^3 \beta_b^2 c^2} \left( \frac{\partial \phi}{\partial x} \right) \]
\[ y'' + \kappa_y y = -\frac{q}{m \gamma_b^3 \beta_b^2 c^2} \left( \frac{\partial \phi}{\partial y} \right) \]

where

\[ m, q \quad \text{particle mass, particle charge} \]
\[ \gamma_b, \beta_b \quad \text{relativistic factors} \]
\[ c \quad \text{speed of light in vacuum} \]
\[ \kappa_x, \kappa_y \quad \text{lattice focusing functions} \]
\[ t \quad d/ds \]

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi = -\frac{q}{\epsilon_0} n = -\frac{q}{\epsilon_0} \int d^2 x'_\perp f_\perp
\]
Core-Particle Model --- Transverse particle equations of motion for a test particle moving inside and outside a uniform density elliptical beam envelope

\[ x'' + \kappa_x x = \frac{2Q F_x}{(r_x + r_y)r_x} x \]
\[ y'' + \kappa_y y = \frac{2Q F_y}{(r_x + r_y)r_y} y \]

Where \( F_x = F_y = 1 \) inside the beam and outside the beam

\[ F_x = (r_x + r_y) \frac{r_x}{x} \text{Re}[\tilde{S}] \]
\[ F_y = -(r_x + r_y) \frac{r_y}{y} \text{Im}[\tilde{S}] \]

with

\[ \tilde{S} \equiv \frac{\tilde{z}}{r_x^2 - r_y^2} \left[ 1 - \sqrt{1 - \frac{(r_x^2 - r_y^2)}{\tilde{z}^2}} \right] \]
\[ = \frac{1}{2\tilde{z}} \left[ 1 + \frac{1}{2} \frac{r_x^2 - r_y^2}{\tilde{z}^2} + \frac{1}{8} \frac{(r_x^2 - r_y^2)^2}{\tilde{z}^4} + \cdots \right] \]
\[ \tilde{z} = x + iy \quad i = \sqrt{-1} \]
Core-Particle Model --- Features of Mathematica based code

Core Beam:
- Quadrupole and Solenoid (Larmor Frame) lattices
- Matched envelope specified specified by
  - Lattice: Lattice Period $L_p$, Occupancy $\eta$, Phase Advance $\sigma_0$
  - Beam: Depressed Phase Advance $\sigma$, Emittance $\varepsilon = \varepsilon_x = \varepsilon_y$
- Envelope can be mismatched (not used in this study)
  - Pure Breathing or Quadrupole mode mismatches or any mixture

Particle Launching:
- Generally launched in groups to probe classes of initial conditions by
  - Phase in lattice period
  - Position: specified relative to matched envelope in annular elliptical regions, on-axis, .....

Diagnostics (for each launching group and in scaled and unscaled coordinates):
- Particle Orbits
- Stroboscopic Poincare phase-spaces
- Single particle emittances
- Frequencies
Particles oscillating radially outside the beam envelope will experience oscillating nonlinear forces that vary with space-charge intensity and can drive resonances.

**Continuous Focusing Axisymmetric Beam Radial Force**

- Nonlinear force transition at beam edge larger for strong space-charge
- Matched beam edge oscillations result in an oscillating nonlinear force while the particle is outside the beam envelope
Core-particle simulations: Poincare plots illustrate resonances associated with higher-order halo production near the beam edge for quadrupole transport

- High order resonances near the core are strongly expressed
- Resonances stronger for higher $\sigma_0$ and stronger space-charge
- Can overlap and break-up (strong chaotic transition) allowing near-core particles to rapidly transport to large amplitudes

### Stable

$\sigma_0 = 100^\circ, \sigma/\sigma_0 = 0.67$

### Unstable

$\sigma_0 = 100^\circ, \sigma/\sigma_0 = 0.1$
Core-particle simulations: Particles launched near the core can become pumped to large amplitude and exhibit strong resonance Poincare phase-space structures.

\[ \sigma_0 = 60^\circ, \ \sigma/\sigma_0 = 0.1 \]

\[ \sigma_0 = 110^\circ, \ \sigma/\sigma_0 = 0.1 \]
Core particle simulations: Large single particle emittance growths for halo particles reinforce that large statistical (rms) emittance growths for the beam are possible if significant number of halo particles are excited.

Single particle emittance:

\[ \epsilon_x = \sqrt{\frac{x}{r_x}}^2 + \left( \frac{xx' - x'x}{\varepsilon_x} \right)^2 \]

\[ \epsilon_x = 1 \implies \text{Beam Edge Particle} \]

\[ \sigma_0 = 60^\circ, \quad \sigma/\sigma_0 = 0.1 \]

\[ \sigma_0 = 110^\circ, \quad \sigma/\sigma_0 = 0.1 \]
Core-Particle simulations: Frequency spectrum of halo particles help define resonance structure

\[ \sigma_0 = 60^\circ, \quad \sigma / \sigma_0 = 0.1 \]

\[ \sigma_0 = 110^\circ, \quad \sigma / \sigma_0 = 0.1 \]

Test Particle x-Frequencies: Initial \( r \in [1.1, 1.2] \)

- Broad and \( > 0.1 \times 60^\circ = 6^\circ \) due to focusing outside of beam being stronger
- Locked at \( \sim 90^\circ \) showing strong 4:1 lattice resonance
Core particle simulations: Stability boundary data from a new “halo” stability criterion are in rough agreement with experimental observations for quadrupole transport

- Start at a point \((\sigma_0, \sigma)\) deep within the stable region
- While increasing \(\sigma_0\) vary \(\sigma\) to find a point (if it exists) where initial launch groups \([1.05, 1.10]\) outside the matched beam envelope are pumped to max amplitudes of 1.5 times the matched envelope
  - Boundary position relatively insensitive to specific group and amplitude growth choices

![Stability Boundary Diagram]

Lagniel also carried out a halo analysis of transport limits in periodic channels but seems to conclude (overly restrictive) that AG transport is unstable for 

Poincare plots generated from self-consistent PIC simulations where particles that evolve out of the core are tagged verify qualitatively the existence of near-core halo properties analyzed that lead to instability.

Lattice period Poincare strobe

\[ \sigma_0 = 110^\circ, \frac{\sigma}{\sigma_0} = 0.2 \]

**Semi-Gaussian**

On axis particles accumulated to generate clearer picture

- Including off axis particles does not changes basic conclusions
Discussion: High phase advances

For quadrupole transport in HIF linacs we avoid space-charge, envelope, and centroid instabilities by operating with $\sigma_0 < 85^\circ$ -- but close to the stability limit:

- Acceptable design condition, but desirable to understand limits better
  - Space-charge limit likely shifts with mismatch (not understood)
  - Stronger focusing can give more compact (cheaper) lattices
- Researchers have suggested that quadrupole transport space-charge limits should also occur for $\sigma_0 < 85^\circ$ (e.g., near $\sigma_0 = 60^\circ$)
  - Appears wrong in self-consistent simulation and in core-particle model

Operation with $\sigma_0 > 85^\circ$ may be relevant in present and future machines:

- Matching sections that compress radial beam envelope have high effective $\sigma_0$
- Future HEDP machines using bunch compression in heavy ion rings are proposing concepts to operate in this regime
  - GSI (Germany) planned facility
  - RIKEN and KEK (Japan) proposals
- Little experimental data beyond SBTE results
  - CERN study by Cappi (fast rotation in a ring) may have observed limit

Limits likely different for solenoidal focusing systems
Core-particle theory suggests that high occupancy solenoid lattices have more benign phase-advance related space-charge limits than quadrupoles.

Poincare Plots

\[ \sigma_0 = 110^\circ \sigma / \sigma_0 = 0.1 \]

Occupancy \( \eta = 0.75 \)

Particles near the edge can get pumped to higher amplitude by the oscillating external potential of a periodically focused intense beam if the beam edge oscillates beyond some critical amplitude.

- **Solenoid focusing:**
  - Envelope flutter small for high focusing occupancy

- **Quadrupole focusing:**
  - Envelope flutter varies little with occupancy but strongly with \( \sigma_0 \)

Result: solenoid space-charge limits are likely less problematic than quadrupoles:

- Must study more to confirm
- Does **NOT** imply that solenoids are better for high intensity transport
Conclusions

High-order space-charge related emittance growth has long been observed in intense beam transport in quadrupole focusing channels with $\sigma_0 \gtrsim 85^\circ$:

- SBTE Experiment at LBNL (1980s)
- Simulations of various distributions for limited parameters
- Not related to centroid or envelope instabilities and are more restrictive

A core-particle model has been developed that suggests observed transport limits result from a halo like mechanism:

- Near edge particles feel strong, rapidly oscillating nonlinear forces when moving just outside the matched beam envelope
- Drives strong resonance chain that limits at large amplitude resulting in a strongly distorted beam and statistical rms beam emittances
- Common theme: maintain beam edge control for good transport
  - Lack of smooth, self-consistent equilibrium in periodic transport precludes full control
  - Halo particles can be lost and act as driving terms for e-cloud effects in magnets

Work is being continued:

- Quantify solenoidal transport limits and effects of beam mismatch
- Manifestation in matching sections and lattice transitions (high effective $\sigma_0$)
- More complete comparisons with self-consistent PIC simulations