Particle trapping by nonlinear resonances and space charge

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The Problem

*System under consideration:*
Bunch of particles stored in an accelerator

*What we want to study*
The effect of Coulombian repulsion
On the nonlinear motion of one particle

*Application*
Keeping high intensity bunch stored in accelerators
As long as possible without dangerous beam loss
Resonances

The transverse motion is described by

\[ x'' + k_x(s)x = \sum_{i+j=n} a_{ij}(s) x^i y^j \]
\[ y'' + k_y(s)y = \sum_{i+j=n} b_{ij}(s) x^i y^j \]

Transverse betatron motion characterized by “tunes”

\[ k_x(s) \rightarrow Q_x \]
\[ k_y(s) \rightarrow Q_y \]
\[ nQ_x + mQ_y = N \]

Dynamic Aperture
Example with 3rd order resonance

Particle with \( x = 1 \sigma_x, \quad y = 0 \sigma_y \)

Maximum horizontal amplitude

Particle lost
Because beyond DA

\( Q_{y0} = 3.29 \)
Effect of space charge

\[ x'' + k_x(s)x = \sum_{i+j=n} a_{ij}(s) x^i y^j + E_x(x, y, s) \]

The space charge creates a defocusing force

It changes the effective Particle tunes

Particle amplitude grows

Resonance condition Is affected
• Focusing forces act in all 3 directions
• The transverse frequency is faster than the longitudinal
• The density of particles is larger in the bunch center
Motion of a particle in a frozen 2D Gaussian distribution

\[ x'' + \left( \frac{Q_0}{R} \right)^2 x = K \exp\left( -z^2 / (2\sigma_z^2) \right) \frac{x}{r} \left( 1 - e^{-x^2 / (2\sigma)^2} \right) + a_2 x^2 \]

\[ r = \sqrt{x^2 + y^2} \]

The longitudinal motion is parametric

\[ z = z_0 \cos(2\pi q_z n + \delta_z) \]

The perveance is given by the tuneshift

\[ K = 2 \left( \frac{\sigma_x}{R} \right)^2 \left( 2Q_{x0} \Delta Q_x - \Delta Q_{x}^2 \right) \]
Single particle tune and transverse amplitude

At $z = 0$

Bare tune

$3Q_{x0} = 13$

Third order resonance

Maximum tuneshift

$\Delta Q_x = 0.1$

This point is Where the 3rd Order island appears
Space charge and lattice resonances

At $z = 0$ and $Q_{x0} = 4.3365$, $Q_{y0} = 4.29$

$\Delta Q_x = 0.1$

Tunes routine by A.Bazzani
Position of the island at different “z”

- Tune at \( z = 0 \)
- Tune at \( z = 1.3 \)

Graph showing the position of the island with \( Q_x \) on the y-axis and \( x / \sigma_x \) on the x-axis.
Role of the longitudinal position

Periodic crossing of a resonance

Bare tune

Resonance

Periodic crossing of a resonance

G. Franchetti  Coulomb 05
Phase space in 1 synchrotron orbit

Example with synchrotron Osc. In 25000 turns

\[ Q_{x0} = 4.35 \]
\[ Q_{y0} = 3.29 \]
\[ x = 1.6\sigma_x \]
\[ y = 0\sigma_y \]
\[ z_{max} = 3\sigma_z \]
Trapping in stable island

First half of the Synch. Osc.

Second half of the Synch. Osc.

At $z = 0$
Effect of the persistent periodic crossing

\[ Q_{x0} = 4.3341 \quad Q_{y0} = 4.29 \quad Q_{z0} = 10^{-3} \]

Maximum extension of islands
Depends on Qx
Resonance crossing induced halo

Maximum X amplitude reached in $5 \times 10^5$ turns

Here the particle is lost
Tune dynamics in presence of chromaticity

Longitudinal plane

Effective transverse
Tune: independent from $x$

Transverse longitudinal coupling model

$$Q_x = -Q_{x0} \frac{\delta p}{p}$$
Effect of chromaticity on particle stability

Region scanned by the tunes because of the chromaticity only

Particle lost

NO SPACE CHARGE
Interpretation

Tune modulation induced by chromaticity and synch. motion

The time (turns) of permanence is peaked at the edge of the tune scan.
Tune dynamics in presence of chromaticity and Space Charge

\[ Q_x = -Q_{x0} \frac{\delta p}{p} \]

This detuning is independent from the transverse position.
The effect of chromaticity on tune dynamics

The particle crosses the resonance at different longitudinal position according if it loses or gain momentum

When the particle loses momentum, the total tune increases

When the particle gains momentum, the total tune decreases
Effect of the chromaticity on the trapping

The particle distinguishes when it gains or loses momentum:

\[ \frac{\varepsilon_z}{\varepsilon_{z,0}} \]

- Momentum gain
- Momentum loss

\[ Q_z \]

- Momentum loss
- Momentum gain
Phase space representation

First half of the synchrotron oscillation

Second half of the synchrotron oscillation
Effect of chromaticity on transverse tune

At $z = 0 \sigma_z$

At $z = 1.3 \sigma_z$
“Instability” with bare tune below the resonance

- Chromaticity rises the tune curve
- Increase of space charge pushes it down
“Instability” with bare tune below the resonance

- Chromaticity rises the tune curve
- Increase of space charge pushes it down

The island oscillates between “infinity” and $x_{\text{min}}$

Particle trapping may occur if bare tune close enough to the resonance
“Instability” with the bare tune above the resonance

- Chromaticity pushes the tune curve down
- Increase of space charge pushes it more down
“Instability” with the bare tune above the resonance

- Chromaticity pushes the tune curve down
- Increase of space charge pushes it more down

The island oscillates between $x=0$ and “infinity”

Particle trapping is always possible
Halo extension and chromaticity

Halo extension in 5E5 turns

“Instability” for bare tune below the resonance

Particle lost

“Instability” for the bare tune above the resonance

no chromaticity
with chromaticity
Experimental campaign 2002-2003 At the CERN-PS

R. Cappi, G. Franchetti, M. Giovannozzi, I. Hofmann, E. Metral, M. Martini, R. Steerembega ICFA2004
Correlation between beam loss and longitudinal size and length

\[ Q_{x0} = 6.265, \quad Q_{y0} = 6.12 \]

R. Cappi, G. Franchetti, M. Giovannozzi, I. Hofmann, E. Metral, M. Martini, R. Steeremberg  ICFA2004
Proof of principle Experiment

Space charge equivalent beams

Coasting beam

Bare tune

Resonance

Periodic crossing
Of resonance is absent!!
NO BEAM LOSS

Bunched beam

Bare tune

Resonance

Periodic crossing!!
BEAM LOSS
SIS18 Resonances


G. Franchetti  Coulomb 05
Experimental setting

Ion: Ar+10
Energy: 11.4 MeV/u
Chopp. Wind: full accept.
Storage time: 1 second
No loss during RF capt.
Momentum spread ~ 1.5e-3

Area for setting tunes
Measurements: beam loss

\[ Q_{x0} = 4.22, \quad Q_{y0} = 3.36 \quad B_f = 0.2 \]

G. Franchetti, A. Franchi, L. Groening, A. Redelsbach
Measurements: transverse beam sizes

Beam size does not change

G. Franchetti, A. Franchi, L. Groening, A. Redelsbach
Conclusion

Space charge in a bunch stored in a nonlinear lattice may cause periodic crossing of a resonance. The presence of chromaticity makes the vicinity of the resonance “unstable”

We found an experimental evidence (preliminary results) which is consistent with the theoretical modeling:

1. Beam loss are significantly enhanced in a bunched beam with respect to those in a “space charge” equivalent coasting beam
2. Transverse beam size in the bunch is not significantly affected by beam loss
References

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• Space charge and octupole driven resonance trapping observed at the CERN proton synchrotron, Phys. Rev. ST Accel. Beams 6, 124201 (2003)
• Beam loss modeling for the SIS100, G. Franchetti and I. Hofmann EPAC 2004
• A. Bazzani 33rd ICFA2004 workshop
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• Particle trapping during passage through a high order resonance, A.W. Chao, M. Month, NIM 121 (1974) 129-138
• Dynamic beam cleaning by a nonlinear resonance A. W. Chao and M. Month NIM 133 (1976) 405-408