Ion acceleration in spherical nanoplasmas

Luís O. Silva

GoLP
Instituto de Plasmas e Fusão Nuclear
Instituto Superior Técnico
Lisbon, Portugal
Acknowledgments

**F Peano, M Marti, RA Fonseca**
GoLP/Instituto de Plasmas e Fusão Nuclear
Instituto Superior Técnico
Lisbon, Portugal

**F Peinetti, R Mulas, G Coppa**
Politecnico di Torino
Torino, Italy

**I Last, J Jortner**
School of Chemistry, Tel-Aviv University,
Tel-Aviv, Israel
Contents

Challenges in numerical modeling of ion acceleration
- Relativistic binary collisions

Ion acceleration in spherical nanoplasmas
- Hydrodynamic expansion vs Coulomb explosion
- Phase space control of ion acceleration with multiple pulses

Direct ion acceleration with chirped pulses
- Towards monoenergetic laser accelerated ion/muon beams

Summary
Solving Maxwell’s equations on a grid with self-consistent charges and currents due to charged particle dynamics

*State-of-the-art*

\[ \sim 10^{10} \text{ particles} \]
\[ \sim (1000)^3 \text{ cells} \]

RAM \sim 1 \text{ Gbyte} - 5 \text{ TByte}
Run time: hours to months
Data/run \sim \text{ few MB} - 10\text{s TByte}

One-to-one simulations of plasma based accelerators & cluster dynamics
Weibel/two stream instability in astrophysics, relativistic shocks, fast igniton/inertial fusion energy, low temperature plasmas

Particle-in-cell (PIC) - (Dawson, Buneman, 1960’s)
Maxwell’s equation solved on simulation grid
Particles pushed with Lorentz force

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Challenges in ion acceleration PIC simulations

**Distinct temporal & spatial scales**
- Very large runs
- Very long runs
  - Optimization for large machines/novel architectures
  - Higher order particle schemes/novel field solvers

**Isolated targets**
- Unbalanced simulation load
  - Dynamic Load Balancing
- Boundary conditions
  - PMLs

**New physics**
- Ionization + Radiation
- Nuclear Physics
- Collisionality
  - Tunneling/annihilation/radiation damping
  - Relativistic MC Collisions

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New Features in v2.0

- Bessel Beams
- Binary Collision Module
- Tunnel (ADK) and Impact Ionization
- Dynamic Load Balancing
- PML absorbing BC
- Optimized higher order splines
- Parallel I/O (HDF5)
- Boosted frame in 1/2/3D

osiris framework

- Massively Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium
  ⇒ UCLA + IST

Ricardo Fonseca: ricardo.fonseca@ist.utl.pt
Frank Tsung: tsung@physics.ucla.edu

http://cfp.ist.utl.pt/golp/epp/
http://exodus.physics.ucla.edu/
## Particle-based kinetic algorithms

### Collisionless

<table>
<thead>
<tr>
<th>Nonrelativistic</th>
<th>Collisionless</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low density – No radiation</strong></td>
<td><strong>High density – No radiation</strong></td>
</tr>
<tr>
<td>- Particles: Vlasov</td>
<td>- Particles: Boltzmann</td>
</tr>
<tr>
<td>- Fields: Poisson</td>
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</tr>
<tr>
<td>- Technique: Particle-in-cell (PIC)</td>
<td>- Technique: PIC + Monte Carlo (MC)</td>
</tr>
<tr>
<td>- Status: established</td>
<td>- Status: partially explored</td>
</tr>
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</table>

### Relativistic

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<tr>
<td><strong>Low density – Radiation</strong></td>
<td><strong>High density – Radiation</strong></td>
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</tr>
<tr>
<td>- Status: established</td>
<td>- Status: almost unexplored</td>
</tr>
</tbody>
</table>
Relativistic kinetic theory

\[ \Delta N = A(v_a, v_b) f_a(x, p_a, t) f_b(x, p_b, t) \Delta p_a \Delta p_b \Delta x \Delta t \]

- \( p_a \): momentum
- \( v_a \): velocity
- \( f_a \): distribution function
- \( p_b \): momentum
- \( v_a \): velocity
- \( f_a \): distribution function

\( A(v_a, v_b) \) determines collision probability
Relativistic invariance of $\Delta N$

**Invariants**
- number of collisions: $\Delta N$
- distribution functions: $f_a, f_b$
- space-time element: $\Delta x \Delta t$

**Invariance of** $A(v_a, v_b) \Delta p_a \Delta p_b$

- $A(v_a, v_b)$ transforms as
  
  $$(\gamma_a \gamma_b)^{-1} = \sqrt{(1-v_a^2)(1-v_b^2)}$$

**General expression for** $A(v_a, v_b)$

$$A(v_a, v_b) = \nu_r \sigma(\nu_r)(1 - v_a \cdot v_b)$$

**Physical constraints**
- $A(v_a, v_b)$ cannot be a function of a single invariant parameter (e.g., $\nu_r$)
- Qualitative difference with nonrelativistic case $A_{NR} = \sigma(|v_a - v_b|)|v_a - v_b|$
Effects of $\Delta N$-invariance violations

Collisions probability of particle pairs affected even with constant $v_r \sigma (v_r)$

- **Collision likely**
  - $1 - v_a \cdot v_b \approx 2$
  - $v_a \approx 1$, $v_b \approx 1$

- **Collision unlikely**
  - $1 - v_a \cdot v_b \approx 0$
  - $v_a \approx 1$, $v_b \approx 1$

Violations of constraints on $A (v_a, v_b)$ break the invariance of $\Delta N$

Unphysical results

Qualitatively wrong equilibrium distributions and energy-temperature relations!

F. Peano et al, PRE (R) 2009
Collision loop for relativistic codes

Consistent MC algorithm for relativistic particle codes with binary collisions

Pair sampling
Choose colliding pairs according to $v_r \sigma(v_r)$
(e.g. $v_r \sigma(v_r) = \text{Constant}$)

New momenta
Update momenta conserving energy and momentum
(e.g. rotation in center-of-mass frame)

Collision output
Determine collision angles using the differential cross section

$\Delta N$ invariance
Reject/accept colliding pairs according to $1 - v_a \cdot v_b$

Jüttner distribution obtained in all energy ranges independently of cross section

F. Peano et al, PRE (R) 2009

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3D ultrarelativistic equilibrium

**MC simulation w/ Osiris 2.0**
- Monoenergetic initial distribution:
  \[ f(x, p, t = 0) \propto \delta[\gamma(p) - \gamma_0], \quad \gamma_0 = 10^4 \]
- 2 x 10^8 computational particles
- \( A(v_a, v_b) \propto (1 - v_a \cdot v_b) \)
- \( v_r \sigma(v_r) = \text{Constant} \)
- **Correct** \( A(v_a, v_b) \propto (1 - v_a \cdot v_b) \)
- **Incorrect** \( A(v_a, v_b) = \text{Constant} \)

**Key results**
- Excellent agreement with theory over several order of magnitudes
- Energy-momentum conservation alone
- \( \Delta N \) invariance
- Energy-momentum conservation alone
- **Jüttner**
- **Modified Jüttner**

**Energy spectrum**
- **Jüttner**
  \[ \rho(\gamma) = \gamma \sqrt{\gamma - 1} \exp\left(-\frac{\epsilon_0 \gamma}{k_B T}\right) \]
  \[ k_B T = 3.33 \times 10^3 \epsilon_0 \]
- **Modified Jüttner**
  \[ \rho_{MJ}(\gamma) = \sqrt{\gamma - 1} \exp\left(-\frac{\epsilon_0 \gamma}{k_B T_{MJ}}\right) \]
  \[ k_B T_{MJ} = 5 \times 10^3 \epsilon_0 \]

F. Peano et al, PRE (R) 2009

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- Summary
Laser-matter interactions

Laser Technology
- IR Technology
  - Wavelength ~ 1 µm
  - Pulse duration ~ 10 fs – 1 ps
  - Intensity up to $10^{21}$ – $10^{22}$ W/cm²
  - Spot size at focus ~ 20 µm
- VUV/X-ray Technology
  - Wavelength ~ 1 – 100 nm
  - Pulse duration as low as 10 fs
  - Intensity up to $10^{18}$ – $10^{19}$ W/cm²
  - Spot size at focus ~ 50 nm – 1 µm

Targets
- Gaseous Targets
  - Underdense (transparent)
  - Poor energy absorption
  - Electron acceleration
  - $\omega_{pe} < \omega_0$
  - $n_{pe} < n_{cr}$
  - Tabletop neutron sources
  - X-ray production
- Solid Targets
  - Overdense (opaque)
  - High energy absorption
  - Ion acceleration
  - Fast ignition for nuclear fusion
  - $\omega_{pe} > \omega_0$
  - $n_{pe} > n_{cr}$

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Laser-cluster interaction: outcomes


Atomic and molecular clusters irradiated with ultraintense laser pulses expand and create a hot dense plasma

- x-rays (keV range, long-lived sources)
- Electron heating (multi-KeV electrons)
- Energetic ions (up to several MeV)
- Nuclear fusion and nucleosynthesis
Laser-cluster interaction: sketch

- Electrons heated/expelled
- Ion sphere expands/explodes
- Laser comes in

- \( R_0 \approx 0.1 \text{–} 100 \text{ nm} \)
- \( n_0 \approx 10^{22} \text{–} 10^{23} \text{ cm}^{-3} \)
- \( I \approx 10^{15} \text{–} 10^{22} \text{ W/cm}^2 \)
- \( \lambda_0 \approx 1 \text{ \mu m} \)
- Spot size \( \approx 20 \text{ \mu m} \)
- \( \tau_{\text{laser}} \approx 10 \text{–} 100 \text{ fs} \)
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- Summary
Expansion or explosion?

Cluster size and laser strength determine the expansion regime.

**Hydrodynamic expansion**
- Electrons remain in the cluster
  - \( \xi_e \ll \delta_e \ll R_0 \)

**Coulomb explosion**
- Electrons escape
  - \( \xi_e \gg \delta_e \gg R_0 \)

Cluster radius \((R_0)\)

- \( \delta_e = \frac{c}{\omega_e} \) = electron skin depth
- \( \xi_e \approx \frac{ca_0}{\omega_0} \) = excursion length
- \( a_0 = \frac{eA_0}{m_ec^2} \propto \sqrt{I} \)

Laser strength \((a_0)\)

- 0.01 to 10

Electron kinetic energy

Coulomb energy

Transition occurs when increasing electron energy [F. Peano et al., Phys. Rev. Lett. 96, 175002 (2006)].
Expansions can be controlled with properly shaped laser pulses.

Strong influence of electron dynamics on the expansion\textsuperscript{1,2}

Laser features determine amount of energy delivered to electrons

Applications

- production of intracluster nuclear reactions in large-scale shock shells\textsuperscript{3}
- single-shot, three-dimensional imaging of biological with x-ray diffraction\textsuperscript{4}

The Vlasov-Poisson (VP) model

### cgs units

\[
\begin{align*}
\frac{\partial f_e}{\partial t} &= -\mathbf{v} \cdot \frac{\partial f_e}{\partial \mathbf{r}} - \frac{e}{m} \frac{\partial \Phi}{\partial \mathbf{r}} \cdot \frac{\partial f_e}{\partial \mathbf{v}} \\
\frac{\partial f_i}{\partial t} &= -\mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{r}} + \frac{Ze}{M} \frac{\partial \Phi}{\partial \mathbf{r}} \cdot \frac{\partial f_i}{\partial \mathbf{v}} \\
\nabla^2 \Phi &= 4\pi e \left( \int f_e d\mathbf{v} - Z \int f_i d\mathbf{v} \right)
\end{align*}
\]

### Dimensionless units

\[
\begin{align*}
\frac{\partial \hat{f}_e}{\partial \hat{t}} &= -\hat{\mathbf{v}} \cdot \frac{\partial \hat{f}_e}{\partial \hat{\mathbf{r}}} - \frac{\partial \hat{\Phi}}{\partial \hat{\mathbf{r}}} \cdot \frac{\partial \hat{f}_e}{\partial \hat{\mathbf{v}}} \\
\frac{\partial \hat{f}_i}{\partial \hat{t}} &= -\hat{\mathbf{v}} \cdot \frac{\partial \hat{f}_i}{\partial \hat{\mathbf{r}}} + \frac{Z_{\text{m}}}{M} \frac{\partial \hat{\Phi}}{\partial \hat{\mathbf{r}}} \cdot \frac{\partial \hat{f}_i}{\partial \hat{\mathbf{v}}} \\
\nabla^2 \hat{\Phi} &= 4\pi \left( \int \hat{f}_e d\hat{\mathbf{v}} - Z \int \hat{f}_i d\hat{\mathbf{v}} \right)
\end{align*}
\]

### Initial conditions

\[
\hat{f}_{i0}(\hat{\mathbf{r}}, \hat{\mathbf{v}}) = \frac{3}{4\pi} \delta(\hat{\mathbf{v}}) \Theta(1 - |\hat{\mathbf{r}}|)
\]
\[
\hat{f}_{e0}(\hat{\mathbf{r}}, \hat{\mathbf{v}}) = \frac{3}{4\pi} \left( \frac{1}{2\pi \hat{T}_0} \right)^{3/2} \times \exp \left( -\frac{\hat{\mathbf{v}}^2}{2\pi \hat{T}_0} \right) \Theta(1 - |\hat{\mathbf{r}}|)
\]

### Dimensionless parameters

- **ion-to-electron charge-to-mass ratio:**
  \[\frac{Z}{M}\]

- **thermal-to-Coulomb energy ratio:**
  \[\hat{T}_0 = \frac{R_0 k_B T_0}{e^2 N_0} = 3 \frac{\lambda_D^2}{R_0^2}\]

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The ergodic model

Assumptions

- Spherical symmetry | cold ions | initial electron distribution known (e.g., Maxwellian)
- Electron mass $\ll$ ion mass $\Rightarrow$ electrons dynamics is sequence of equilibria
- Equilibrium configurations characterized by **ergodic distribution**: $Q(r, \epsilon; \{\Phi\}) = \frac{r^2 [\epsilon + e\Phi(r)]^{\frac{1}{2}}}{\int r'^2 [\epsilon + e\Phi'(r')]^{\frac{1}{2}} dr'}$

Equations

- Ion trajectory: $r_i (r_0, t)$
- Electron energy: $\epsilon (\epsilon_0, t)$
- Ion density: $n_i (r, t)$
- Electron density: $n_e (r, t)$
- Electrostatic potential: $\Phi (r, t)$

$$\begin{align*}
\frac{\partial^2 r_i}{\partial t^2} &= -\frac{Ze}{M} \frac{\partial \Phi}{\partial r} (r_i) \\
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right) &= 4\pi e (n_e - Z n_i) \\
n_i (r_i) &= n_{i,0} (r_0) \left( \frac{r_0}{r_i} \right)^2 \\
n_e &= \frac{1}{4\pi r^2} \int \rho_{e,0} (\epsilon_0) Q(r, \epsilon; \{\Phi\}) d\epsilon_0 \\
\frac{d\epsilon}{dt} &= -e \frac{\partial \Phi}{\partial t} (r, t) Q(r, \epsilon; \{\Phi\}) dr \\
\text{electron energy spectrum: } \rho_e (\epsilon, t) d\epsilon &= \rho_{e,0} (\epsilon_0) d\epsilon_0
\end{align*}$$

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Expansion dynamics (low $\hat{T}_0$)

\[ \hat{T}_0 = 7.2 \times 10^{-3} (\lambda_{D,0}/R_0 = 4.9 \times 10^{-2}) \]
Expansion dynamics (low $\hat{T}_0$)

$$\hat{T}_0 = 7.2 \times 10^{-3} \left( \lambda_{D,0}/R_0 = 4.9 \times 10^{-2} \right)$$

Electron and ion density

Ion phase-space profile
Expansion dynamics (high $\hat{T}_0$)

$$\hat{T}_0 = 7.2 \times 10^{-2} (\lambda_{D,0}/R_0 = 1.5 \times 10^{-1})$$

Electron and ion density

Ion phase-space profile
Expansion dynamics (high $\hat{T}_0$)

$$\hat{T}_0 = 7.2 \times 10^{-2} (\lambda_{D,0}/R_0 = 1.5 \times 10^{-1})$$
Transition to Coulomb explosion

- transition to monotonic energy spectra
- onset of the Coulomb explosion regime

Features of final ion spectrum described by simple laws

Shape of asymptotic spectra

Spectral features vs. $\hat{T}_0$


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- Summary
The double-pump technique

1. 1st pulse: slow expansion ➔ decreasing density profile
2. 2nd pulse: abrupt explosion ➔ shock-shell formation

Shock shell formation and evolution

Nonuniform density profile

- force maximum within cluster
- inner ions overtake outer ions

multi-branched profiles in phase space

vertical tangent ➔ density singularity (caustic)
horizontal tangent ➔ spectrum singularity

Spatial trajectories of sample ions
(color indicates energy: blue → yellow → red)
Double pump: ion dynamics
Intracluster nuclear reactions

Number of reactions per unit time and volume

\[ \mathcal{R} = \frac{1}{2} \sum_{i,j=1}^{3} n_i(r)n_j(r)\sigma(\|v_i - v_j\|)|v_i - v_j| \]

\( n_i \) = density on ith branch
\( \sigma \) = cross section

N. intracluster reactions

\[ \mathcal{N} = 4\pi \int_{t_{sh}}^{\infty} \int_{r_{sh}}^{R_{sh}} \mathcal{R} r^2 dr dt \]

For large D/DT clusters

intracluster and intercluster fusion yields are comparable\(^1\)
Intracluster reactions demonstrated with 3D Molecular Dynamics (scaled electron and ion dynamics (SEID) technique)

Cluster radius: $R_0 = 110$ nm

<table>
<thead>
<tr>
<th>Homonuclear: deuterium (double pump)</th>
<th>Heteronuclear: deuterium-tritium (single pump)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First pulse</strong></td>
<td><strong>Laser pulse</strong></td>
</tr>
<tr>
<td>• intensity: $10^{15} - 10^{17}$ W/cm²</td>
<td>• intensity: $10^{20}$ W/cm²</td>
</tr>
<tr>
<td>• duration: 20 – 30 fs</td>
<td>• duration: 20 fs</td>
</tr>
<tr>
<td><strong>Second pulse</strong></td>
<td><strong>Homogeneous mixtures</strong></td>
</tr>
<tr>
<td>• intensity: $10^{20}$ W/cm²</td>
<td>• reactions: up to $N = 1$</td>
</tr>
<tr>
<td>• duration: 20 fs</td>
<td></td>
</tr>
<tr>
<td>• delay: 110 – 180 fs</td>
<td><strong>Layered clusters (D core + T shell)</strong></td>
</tr>
<tr>
<td><strong>Reactions: up to $N = 4 \times 10^{-3}$</strong></td>
<td>• reactions: up to $N = 0.5$</td>
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- Summary
Laser-based acceleration

Electrons

- Relativistic intensities ($>10^{18} \text{ W/cm}^2$, IR)
  achievable with modern lasers

- Several schemes with single or multiple laser pulses

Ions

- Relativistic intensities ($>10^{24} \text{ W/cm}^2$, IR)
  beyond present technology

- All-optical acceleration with nonrelativistic intensities
  (works also with muons)

- Several schemes exploiting electrostatic plasma waves

- Several schemes exploiting electrostatic fields in solid targets
Acceleration principle

**Concept**

- Counterpropagating lasers drive slow ponderomotive beat wave
- Frequency chirp induces variation in beat-wave phase velocity
- Particles are trapped and continuously accelerated

**EM wave 1**
- coherent
- polarized
- nonrelativistic \( \left( \frac{qA_1}{Mc^2} \ll 1 \right) \)
- chirped

**Heavy Particles**
- charge: \( q \)
- mass: \( M \)

**EM wave 2**
- coherent
- polarized
- nonrelativistic \( \left( \frac{qA_1}{Mc^2} \ll 1 \right) \)
- chirped

Acceleration principle

- laser 1 (chirped)
- laser 2 (fixed frequency)
- beat wave
- ponderomotive beat wave
Acceleration principle

- laser 1 (chirped)
- laser 2 (fixed frequency)
- beat wave
- ponderomotive beat wave
**Field configuration**

### EM wave 1

\[
\mathbf{A}_1 = A_1 \cos(\theta) \sin[\Phi_1(\xi_1)] \hat{e}_y \\
+ A_1 \sin(\theta) \cos[\Phi_1(\xi_1)] \hat{e}_z
\]

\[\xi_1 = x - ct\]

\[k_1(x, t) = \Phi'_1, \quad \omega_1(x, t) = \Phi'_1\]

### EM wave 2

\[
\mathbf{A}_2 = A_2 \cos(\theta) \sin[\Phi_2(\xi_2)] \hat{e}_y \\
+ A_2 \sin(\theta) \cos[\Phi_2(\xi_2)] \hat{e}_z
\]

\[\xi_2 = -x - ct\]

\[k_2(x, t) = -\Phi'_2, \quad \omega_2(x, t) = \Phi'_2\]

### Slow beat wave

\[
k(x, t) = \frac{1}{2}(\Phi'_1 + \Phi'_2)
\]

\[
\omega(x, t) = \frac{c}{2}(\Phi'_1 - \Phi'_2)
\]

\[
v_{\phi}(x, t) = \frac{\omega}{k} = c\frac{\Phi'_1 - \Phi'_2}{\Phi'_1 + \Phi'_2}
\]

### Fast beat wave

\[
K(x, t) = \frac{1}{2}(\Phi'_1 - \Phi'_2)
\]

\[
\Omega(x, t) = \frac{c}{2}(\Phi'_1 + \Phi'_2)
\]

\[
V_{\phi}(x, t) = \frac{\Omega}{K} = c\frac{\Phi'_1 + \Phi'_2}{\Phi'_1 - \Phi'_2}
\]

---

**Initial beat wave velocity = initial ion velocity**

\[
v_{\phi}(0, 0) = c\beta_0 \quad \Rightarrow \quad \frac{\omega_{02}}{\omega_{01}} = \frac{1 - \beta_0}{1 + \beta_0}
\]
Basic equations

**Equation of motion**

\[
\frac{dp_x}{dt} = -\frac{q^2}{2\gamma Mc^2} \frac{\partial}{\partial x} (A_1 + A_2)^2
\]

- average over fast oscillations
- normalization

**Ponderomotive equation**

\[
\frac{d\hat{p}_x}{d\hat{t}} = -\frac{\hat{A}_1 \hat{A}_2}{2\gamma} \frac{\partial}{\partial \hat{x}} \cos (\Phi_1 - \Phi_2)
\]

**Energy equation**

\[
\frac{d\gamma}{d\hat{t}} = \frac{\hat{A}_1 \hat{A}_2}{2\gamma} \frac{\partial}{\partial \hat{t}} \cos (\Phi_1 - \Phi_2)
\]

\[
\hat{A}_1 = \frac{qA_1}{Mc^2} \ll 1 \quad \hat{A}_2 = \frac{qA_2}{Mc^2} \ll 1
\]

\[
\hat{x} = k_0 x \quad \hat{t} = k_0 c t \quad k_0 = k (0,0)
\]

\[
\hat{p}_x = \frac{p_x}{Mc} \quad \gamma^2 = 1 + \hat{p}_x^2 + \hat{A}_1^2/2 + \hat{A}_2^2/2 + \hat{A}_1 \hat{A}_2 \cos (\Phi_1 - \Phi_2)
\]

\[
\Phi_1' - \Phi_2' \neq \text{constant}
\]

\[
\text{Wave-particle energy transfer}
\]
Trapping

Beat-wave (accelerating) frame

Beat-wave trajectory: \( \hat{x}_{\phi_0}(\hat{t}) \) such that
\[
\Phi_1(\hat{x}_{\phi_0} - \hat{t}) - \Phi_2(-\hat{x}_{\phi_0} - \hat{t}) = \phi_0
\]

Phase difference: \( \psi = 2 [\hat{x} - \hat{x}_{\phi_0}(\hat{t})] \)

Proper time: \( d\tau = d\hat{t}/\gamma \)

Inertial force: \( \alpha_{\phi_0}(\tau) = \frac{d^2 \hat{x}_{\phi_0}}{d\tau^2} = \gamma \frac{d}{d\hat{t}} \left( \gamma \frac{d}{d\hat{t}} \hat{x}_{\phi_0} \right) \)

Equation of motion
\[
\frac{d^2 \psi}{d\tau^2} = -\frac{\partial}{\partial \psi} U(\psi, \tau)
\]

Effective potential for trapped particles
\[
U(\psi, \tau) \approx 2\hat{A}_1\hat{A}_2 \cos(\hat{k}\psi + \phi_0) + 2\alpha_{\phi_0}\psi
\]

Necessary condition for trapping
\[
|\alpha_{\phi_0}(\hat{t})| < \hat{A}_1\hat{A}_2\hat{k} \left[ \hat{x}_{\phi_0}(\hat{t}), \hat{t} \right]
\]
inertial force max ponderomotive force
Trapping dynamics and early stages

laser 1 linearly chirped: \( \sigma_1 = -\hat{A}_1 \hat{A}_2 = -3 \times 10^{-5} \)

ions & beat-wave potential

ions & effective potential

\( \text{time} = 0.00 \pm 0.00 \ \omega_0^{-1} \)

kinetic energy = 0.00E+00 MeV
laser 1 linearly chirped: $\sigma_1 = -\hat{A}_1 \hat{A}_2 = -3 \times 10^{-5}$
Scalings and laser requirements

**Scalings for nonrelativistic regime** (good for heavy ions)

In relativistic regime scalings depend on the specific chirp laws

Large variations in velocity require large excursions in frequency

### Acceleration distance

\[ \Delta x [\mu m] \approx 6 Z_p^2 A_p^{-2} I_1^{1/2} [10^{20} \text{W/cm}^2] I_2^{1/2} [10^{20} \text{W/cm}^2] \lambda_{01}^{-1/2} [\mu m] \lambda_{02}^{-1/2} [\mu m] \Delta T^2 [\text{ps}] \]

### Maximum energy gain

**vs. laser intensity & time**

\[ \Delta \mathcal{E}_M [\text{MeV}] \approx 0.8 Z_p^3 A_p^{-3} I_1 [10^{20} \text{W/cm}^2] I_2 [10^{20} \text{W/cm}^2] \lambda_{01} [\mu m] \lambda_{02} [\mu m] \Delta T^2 [\text{ps}] \]

**vs. laser parameters**

\[ \Delta \mathcal{E}_M [\text{MeV}] \approx 0.08 Z_p^4 A_p^{-4} \mathcal{E}_1 [\text{J}] \mathcal{E}_2 [\text{J}] w_{01}^{-2} [\mu m] w_{02}^{-2} [\mu m] \lambda_{01} [\mu m] \lambda_{02} [\mu m] \]

\[ = 8 \times 10^{-7} Z_p^4 A_p^{-4} \mathcal{E}_1 [\text{J}] \mathcal{E}_2 [\text{J}] Z_{R1}^{-1} [\text{mm}] Z_{R2}^{-1} [\text{mm}] \]

### Frequency excursion required

Large variations in velocity require large excursions in frequency

\[ \Delta \omega_1 = \frac{2 (\beta - \beta_0)}{1 - \beta} \]

\[ \Delta \omega = \beta - \beta_0 \]

\[ \Delta \omega_2 = \frac{2 (\beta_0 - \beta)}{1 + \beta} \]
Proton acceleration in a plasma

peak intensities:
\[ I_1 = 1.3 \times 10^{21} \text{ W/cm}^2 \]
\[ I_2 = 8.5 \times 10^{20} \text{ W/cm}^2 \]

chirp coefficient:
\[ \sigma = -2 \times 10^{-5} k_0^2 \]

ref. wavelength:
\[ \lambda_0 = 820 \text{ nm} \]

proton density:
\[ n_i = 5 \times 10^{16} \text{ cm}^{-3} \]

slab thickness:
75 \( \mu \text{m} \)

pulse duration:
4.2 ps

spot size:
10 \( \mu \text{m} \)

longitudinal phase space
kinetic-energy profile

L. O. Silva | Coulomb 09 | Senigallia, June 15 2009
Proton acceleration in a plasma

- **Peak intensities:**
  - \( I_1 = 1.3 \times 10^{21} \text{ W/cm}^2 \)
  - \( I_2 = 8.5 \times 10^{20} \text{ W/cm}^2 \)

- **Chirp coefficient:**
  - \( \sigma = -2 \times 10^{-5} k_0^2 \)

- **Reference wavelength:**
  - \( \lambda_0 = 820 \text{ nm} \)

- **Proton density:**
  - \( n_i = 5 \times 10^{16} \text{ cm}^{-3} \)

- **Slab thickness:**
  - 75 \( \mu \text{m} \)

- **Pulse duration:**
  - 4.2 ps

- **Spot size:**
  - 10 \( \mu \text{m} \)
Multi-stage acceleration for muon factory

Lasers always perpendicular to direction of incoming beam

- No need for phase synchronization
- Large $\Delta \beta$ with limited $\Delta \omega$

Muon acceleration

![Graph showing muon acceleration vs. time and energy](image)
Summary and next steps

- **Challenges in modeling ion acceleration in realistic conditions with PIC simulations is driving exciting progresses**
  - still some surprises even for “standard” techniques when pushed to the limits

- **Detailed phase space control in cluster explosions**
  - also demonstrating alternative directions for laser-thin target interactions

- **Ultra high intensity multiple laser beams allow for ultra fast direct + detailed control of heavy species acceleration**
  - (e.g. muons + isotope separation)