Mobilis in Mobile: a probabilistic and chronotopic model of mobility in urban spaces

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(‡) This contribution was presented at the workshop "Theoretical biology 2: history and present themes" (Arcidosso, Italy, September 1-3, 1999)

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Abstract

In this communication we propose an urban mobility model based on individual random walk driven by a chronotopic action with a deterministic public transportation network. In the absence of chronotopoi are switched on, they attract people according to a given law and we obtain a sort of diffusive motion. The model can describe many different kinds of dynamical systems, including biological ones. The work is in progress and the next step will be an empirical test in a concrete case.

Introduction

Since sixties the problem of building physical mathematical models useful to study the dynamics of cities growth, the social transformations and citizens mobility was posed. More precisely such models have been developed in order to explain the whole structure of cities as abstractly, usually, at that time, global, centralised and supported, in the best cases, by an illuministic philosophy (esprit). Generally the city was considered as a static
system, i.e. the models simulated the structure at one point in time, assumed to be an equilibrium point, and ignored the time dependent evolution because two main difficulties arose: first, from an empirical point of view, it is hard to collect sufficiently long time series of data and, second, the dynamical equations are intrinsically non linear and usually we cannot solve them by analytic methods. In short the urban modelling was global, macroscopic and static, or stationary, with different approaches, dynamical (gravitational models), thermodynamical (entropic models) or, especially in USA, based on the theory of games and the Lotka-Volterra type equations (for urban policies) (Besussi, Cecchini, eds. [1996]). In the seventies a new scenario is opened both because the large scale models are too non-realistic (Lee [1973]) and because studies on modelling the cities as dynamical systems are beginning (Batty [1971]). Contemporarily a great development in the mathematical and physical studies on the non linear dynamical and complex systems, both deterministic and stochastic, emerges (Nicolis, Prigogine [1989]). The reciprocal influence between deterministic orbits and stochastic perturbations are important for the definition of a stationary state and for the diffusion equations of the system. This new approach, someone speaks of a new paradigm, allowed to construct models which cover not only the physical phenomena, but also biological, economic, social ones and planning (Hutchinson and Batty, eds.[1986]). Moreover, in the specific context of urban studies and planning, the free actions of the single and the options of associated citizens at different scales, from the single street and quarter to the city, become relevant both at theoretical and practical level, because the perception of urban spacetime varies in each individual and in each social group and planning is thought more as a harmonising tool from local to global than a centralised governing (obviously not everywhere and not by all people) (Cecchini [1996]). The elements of decision are based on citizen’s participation that becomes a vital part of the planning process that must be sensible to the rapidly evolving individual and social behaviours. In this perspective it is adequate to build flexible and local models as tools for the models able to describe, explain, analyse the features of the system and able to predict its evolution or the evolution of some observable that are defined as meaningful. In this framework mathematical objects such as cellular automata and neural networks (Batty, Xie [1996]) find a natural application to the cities. Moreover the exponential growth of the computers performance and the creation of new object oriented programming languages as C++, and suitable graphic interfaces, have opened the possibility to experience very complex systems by using the computer as a virtual laboratory in which experiments are performed. A theoretical guide is needed anyhow and the results of the model must be validated by real experiences and empirical observations (Turchetti [1999]). So the hypothesis of modelling some urban phenomena is, in our opinion, not utopian and we propose here an evolutive model of mobility network, inspired by the time dependent physics of the city (Bonfiglioli and Mareggi, eds. [1997]), using statistical techniques and simulations. The model takes into account the mobility of the single citizen, the flow variations of urban populations during different intervals of time, for example a day, a month, a year, with the constraints of the biological rhythms of human beings, the urban chronotopic structure and the morphology of the city. The algorithmic implementation on a computer should allow to study the behaviour of one or more critical control parameters, considering our virtual mobility city as a complex self-organising system with possible
phase transitions between different regimes. Moreover we want to underline that the model could consider the tendencies and the requests of mobility, a sort of “potential mobility”, using the Bayes-de Finetti concept of probability (de Finetti [1974]). Finally this program of research is performed in the framework of the national project MURST “Tempi Urbani” (1997-1999) (Bazzani, Giorgini, Servizi, Turchetti [1999]) and will continue in the context of the MURST “Qualità della vita nelle metropoli di terza generazione” (1999-2001). In the next pages we describe the model in a more detailed way and we develop the mean field approximation showing the good agreement with simulations. At the end we list some provisional conclusions and the possible perspectives.

The model

Usually the models used in planning to study the urban mobility are the so called origin-destination (O-D) models (Cascetta, Cantarella [1993]). Roughly speaking, two fixed points are defined, the origin and the destination, and one or more path(s) (arc(s)), with an assigned probability, in the deterministic or stochastic cases. The two fundamental variables are the capacity of the streets and the volume of the private and public cars, and the critical parameter is the ratio between them. Usually the values of the variable are chosen at constant times and averaged on time interval in order to obtain the mean behaviour. The models describe an equilibrium macroscopic situation where the dynamical properties of the single citizen are neglected. Therefore this modelling is effective only if the traffic is sufficiently regular and not affected by strong, sudden and unexpected flow changing, i.e. the O-D models are unable to study both the possible turbulent regimes and the traffic jams (time localized phase transitions from fluid to solid state). Moreover they do not take into account the mobility which not necessarily type emerges and becomes important in the urban spaces of third generations metropolis: the so called zigzagging mobility (Mareggi [1997]) which precisely does not depend on two fixed points, the origin and the destination. Furthermore the citizens’ requests of mobility are spread out in every time and period and so we are confronted with a traffic which is not so concentrated in some fixed hours of the day as before, when the mobility was strongly modelled by the rigid time table of working, especially the industrial one. In this situation we want to describe, explain and, if possible, predict the time dependent effects on mobility of individual zigzag paths and fluctuations due to a stochastic interaction between the citizens’ requests of mobility and the offer by the city structures and public transport; at the moment we have neglected the interactions with the private cars. In order to achieve our purposes it is clear that a traditional reductionist point of view appears to lead nowhere and also an overall approach seems to be inadequate due to the lack of knowledge of interactions between the components of the system. So we choose an intermediate philosophy which starts from the behaviour of the individual by using the random walk as a natural candidate for describing the zigzagging mobility.

The underlying idea is that the individual minute details are not able to change drastically the collective evolution if the laws governing the behaviour of components are sufficiently smooth (Parisi [1992]), but some emergent properties which characterize the macroscopic dynamics of our model are essentially due to the presence of a microscopic
dynamics. Moreover we know (Ruelle [1990]) that if we impose to a complex system a simple global condition, the features satisfying this condition are usually characterized by a univocal probabilistic set and the system spontaneously evolves, in a sufficiently long time, to a state of maximal entropy. Now we can specify the principal elements of our model, the urban mobility spacetime, the chronotopoi, the citizens, that, by means of reciprocal interactions, produce the dynamics of the systems.

*The urban mobility spacetime.* It is constituted by:

1) a network of roads crossing each other according to a known map, where the streets are completely characterized through their location, capacity, morphology and relationships (crossing) with other streets;
2) a network of public transport lines where the "trains", characterized through their capacity, run and stop at fixed stations to load and unload people in a deterministic way; the stations are the junctions of interchange between the walking streets and the public transport lines (subway, railway, busway);
3) an absolute scale of time that marks the distance, measured on the streets, between crossings; the distance, measured on public transport lines, between the stations and finally, with the same definition of measure, the distance between both crossings and stations.

*The chronotopoi.* In general we can define the chronotopos as the prime agent of time-dependent urban activities, i.e. the agent that introduces time correlations which we could not expect without (Giorgini [1997]). In planning (Bonfiglioli [1997]) we call chronotopos an area of the town marked with some time functions describing its action with respect to people: for instance we can introduce the chronotopos University that attracts students and professors between 8 hours in the morning and 14 hours in the afternoon, while repelling them after 19. Furthermore inside the chronotopos usually are present attractors, that is one or more spots where some classes of people are expected to go: for example the Physics Department inside the University as attracting physicists.

*The citizen.* We define the citizen as a subject carrying a set of individual properties such as sex, age, job and so on. Depending on these characteristics a citizen is assigned one, or more, propension (Popper [1990]) which makes her/him tending to a suitable chronotopos, and one propension can be strong in a time interval of the day becoming weak during another, and vice versa. There exists also a quota of citizens without any propension. The propension acts as a bias on the probabilities which define the random walk, whose intensity increases with the distance between the individual and the chronotopos (strong propension). Once the citizen reaches the chronotopos neighborhoods and he is not captured by an attractor, the bias is suppressed and the pure random walk comes in the foreground. Moreover each citizen has a memory introduced to avoid possible vicious loops. Anyway it is short enough to keep the system "almost" markovian when the chronotopoi are suppressed. When an individual goes out of the network representing our virtual town, instantaneously another is randomly created, in order to conserve the global number of citizens; we can also take into account periodic fluctuations of populations. Finally, from the geometrical point of view, our citizen, at the moment, has dimension zero.

A computer program simulating our model is under development; it is written in C++ language, taking advantage of the object oriented programming style. All elements of our
model are translated into objects inside the program, thus they can be easily created, modified and destroyed. The program runs as a X11 client under the Unix operating system and can be immediately transported on any computer platform which can support it.

Mean field theory

First (the one state model) we consider a square network of \( N \times N \) streets where \( M \) pedestrians (everyone is a point) stay in the crossings and move randomly. More precisely the state of each individual is defined by a sequence of four possible directions, up, down, left, right. At every time step he moves randomly from a crossing to another of the four possible neighborhoods with the constraint that he cannot return in the same crossing where he was at the previous time step (one step memory). So we obtain a sequence of directions which codes the specific movement of the single citizen step by step, i.e. a sort of genetic, even though it is much more simple than the biological one, code of individual mobility, and we can represent this code as a string, composed by the random succession of four symbols (directions), of length \( k \), where \( k \) is the number of steps between the initial site and the final one. For \( k \gg 1 \) it is possible to show (Doob [1953]) that if the chronotopoi are absent the probability of finding anyone of the four symbols in the string tends to \( 1/4 \). That is pedestrian will visit all the network with the same frequency because of the same probability for all directions, so after \( k \) time steps, with \( k \) large enough, the \( M \) citizen will be uniformly distributed with an exponential relaxing \( \exp(-Dt) \) where \( D \) is the diffusion coefficient. If the chronotopoi are absent i.e. after a transient the mobility is uniform on the network. We superimpose a \( nxn \) public lines network where the means of transport, the trains, run in a deterministic way, moving along a fixed direction from a station to the following one in a time step. Obviously the stations spacially coincide with some streets crossing and these are the junctions between the public transport and the pedestrian networks. Now the citizen moves randomly on the pedestrian network but when he arrives to station he must take a train, becoming a user. So the individual can have two dynamical states, the pedestrian one \( p \) and the user state \( u \). Moreover, if at the station a citizen does not find a train, he must wait for one or more time steps until it comes. Therefore we have also the possible wait state \( w \). Finally we will designate as \( p(t) \) the normalized number of pedestrians (the percentage) at time \( t \), \( u(t) \) and \( w(t) \) respectively for users and waiting. Putting the trains frequency \( f = 1 \), that is one train is present in every station all the time, and with an infinite capacity, we obtain, because \( w(t) = 0 \), a two state model, \( p \) and \( u \). If \( M \) remains constant then

\[
p(t) + u(t) = 1, \quad \forall t
\]

Being \( P_{pu} \) and \( P_{up} \) the probability transitions from \( p \) to \( u \) and vice versa, with \( P_{pu} = (n/N)^2 \) and \( P_{up} = Qq \), where \( Q \) is the probability of descending after a station and \( q \) the probability of going out of the station, the evolution of populations is described by the following equations:

\[
\begin{align*}
p(t+1) &= p(t) + P_{up}u(t) - P_{pu}p(t) \\
u(t+1) &= u(t) - P_{up}u(t) + P_{pu}p(t)
\end{align*}
\]
which, for an equilibrium regime, are solved by

\[ p(t) = p_\infty + (p(0) - p_\infty)(1 - P_{up} - P_{pu})^t \]

with \( p_\infty = P_{up}/(P_{up} + P_{pu}) \) and \( P_{up} \leq P_{pu} \) where the equal sign holds if the user goes down at the first station. In the continuous limit, that is assuming small probability transitions and \( t \gg 1 \), the equations (1) become the differential equation

\[ \frac{dp}{dt} = -(P_{up} + P_{pu})p(t) + P_{up} \]

which reaches the stationary solution \( dp/dt = 0 \) in a relaxing time \( T = 1/(P_{up} + P_{pu}) \). The three states model occurs when \( f \neq 1, w \neq 0 \) and, considering the transition probability from pedestrian to waiting state \( P_{pw} \), and in a similar way for \( P_{uw}, P_{wu} \), the corresponding evolution equations are:

\[
\begin{align*}
  p(t+1) - p(t) &= P_{up}u(t) - P_{pu}p(t) - P_{pw}p(t) \\
  u(t+1) - u(t) &= -P_{up}u(t) - P_{uw}u(t) + P_{pu}p(t) + P_{wu}w(t) \\
  w(t+1) - w(t) &= P_{uw}u(t) + P_{pw}p(t) - P_{wu}w(t)
\end{align*}
\]

with the conservation law \( p(t) + u(t) + w(t) = 1 \). The equations (3) remain linear if the trains capacity is infinite because \( P_{pu} \) and \( P_{wu} \) are constant. In the finite case we have a saturation threshold of the trains, beyond which \( P_{pu} \) and \( P_{wu} \) become zero. At the equilibrium regime we obtain the solutions:

\[
\begin{align*}
  p &= \frac{P_{up}P_{wu}}{R} \\
  u &= \frac{P_{wu}(P_{pu} + P_{pw})}{R} \\
  w &= \frac{(P_{up}P_{pw} + P_{uw}(P_{pu} + P_{pw}))}{R}
\end{align*}
\]

where \( R = P_{up}(P_{pw} + P_{wu}) + (P_{uw} + P_{wu})(P_{pu} + P_{pw}) \).

**Conclusions and perspectives**

The work is in progress and many problems are open. It is evident that many parameters must be carefully tuned in order to match a realistic situation. Among all we point our attention on the following topics and questions.

1) We need a chronotopic map in a concrete urban case, that is spatial feature and realistic time law for every chronotopos. And also a reasonable form for the chronotopic action; at the moment we have chosen an elastic one.

2) How the bias and the neighborhood of a chronotopos could be defined and how they interact with each other.

3) Which individual properties of the citizens are expected to have the greater influence on the mobility.
4) How the interactions both between moving citizens and the chronotopoi are important and could be described.

5) How to introduce the possibility for a citizen to get information on the global state of the mobility in the city.

Moreover our model does not consider the private transports, and so it takes into account only the mobility by walking (slow mobility) and on public transport, but the bus mobility is affected by private traffic and it is not regular as the subway. For this a friction could be introduced in order to reduce the public transports speed (obviously a zero friction corresponds to the subway). In this framework we would perform the following research program.

1) A more accurate exploration of the model in order to investigate the critical parameters and the variables behaviours at very different scales, both studying the possible analytic solutions and simulating on a long range.

2) We have obtained an analytic description of the model evolution in the mean field approximation in good agreement with simulations without chronotopoi; it would be important to find a reasonable mathematical formulation for the system also when the chronotopoi act.

3) We would make some virtual experiments with realistic data (city map, real public transport network and so on).

4) It will be very useful to have the possibility of experimenting the real mobility of a group of citizens monitored by individual GPS detectors and to compare the empirical results with the corresponding simulations. The goal could be to find a reasonable agreement between simulation and real experiment.

Finally it is clear for us that the previous problems and the research program can be performed only in a multidisciplinary context with the contribution of planning scientists, sociologists, architects, engineers and all people that study urban transformations.

Acknowledgements

We are strongly indebted to S. Bonfiglioli for many helpful discussions and criticisms. We would also like to thank the SAT Laboratory, Dip. di Scienze del Territorio, Politecnico di Milano, Italy for the useful discussions and its kind hospitality. This research was financially supported by the project MURST ”Tempi urbani”, Ricerche di Rilevante Interesse Nazionale, Italy (1997-1999).

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