A Chronotopic Model of Mobility in Urban Spaces

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Abstract

In this paper we propose an urban mobility model based on individual stochastic dynamics driven by the chronotopic action with a deterministic public transportation network. Such a model is inspired by a new approach to the problem of urban mobility that focuses the attention to the individuals and considers the presence of random components and attractive areas (chronotopoi), an essential ingredient to understand the citizens dynamics in the modern cities. The computer simulation of the model allows virtual experiments on urban spaces that describe the mobility as the evolution of a non-equilibrium system. In the absence of chronotopoi the relaxation to a stationary state is studied by the mean field equations. When the chronotopoi are switched on the different classes of people feel an attraction toward the chronotopic areas proportional to a power law of the distance. In such a case a theoretical description of the average evolution is obtained by using two diffusion

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equations coupled by local mean field equations.

**Key words:** Mobility, random walk, chronotopos, average equations.

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**Introduction**

Since the sixties the problem of building suitable physics-oriented mathematical models to study the dynamics of cities growth, social transformations and citizens mobility was posed. More precisely such models have been developed in order to explain the whole structure of cities as abstractly rational and to predict the consequences of urban planning. Generally the city was considered as a static system in a stationary equilibrium situation. Different approaches, based on mechanical (gravitational) and thermodynamical (entropic) models or on the theory of games and the Lotka-Volterra type equations for urban policies were proposed(1). In the seventies the scenario changed because the large scale models appeared to be too non-realistic(2) and because modeling the cities as dynamical systems started to be considered(3). At the same time a great development in the mathematical and physical studies on the non-linear dynamical and complex systems, both deterministic and stochastic, emerged(4). In this framework mathematical objects such as cellular automata and neural networks are natural candidates to cities modeling(5). One

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of the main problem in the modern cities is the growing of the traffic congestion due to the increase of the mobility request, so that several deterministic or stochastic models have been proposed to simulate the traffic flow and to help the authorities for a better transportation planning(6). A widely used class of models considers a origin-destination (OD) mobility and focuses the attention to the dynamical equilibrium of the system(7; 8). A common characteristic of these models is that they do not simulate the dynamics of the single individual, but consider each of them bound to a fixed transportation mean (usually the private cars) and look for an optimal equilibrium in a roads network according to the demand of the OD mobility. Other approaches study more specifically the emergence of traffic jams as phase transition phenomenon in a stochastic cellular automata(9). Very recently the urban experts have remarked that the mobility problem in the modern cities is the result of the interaction among different citizen classes (city users, businessmen, housewives....) that have different requests of mobility and can use various transportation means(10). Moreover it has been pointed out the appearance of a new kind of mobility that cannot be reduced to the OD mobility and comes out as a random walk in the vicinity of certain urban areas (zigzagging mobility)(11; 12). In this framework the request of mobility is explained by the existence of chronotopic areas that attract specific citizens categories with a specific time schedule(13). A chronotopos is defined as the prime agent of time-dependent urban activities, i.e. the agent that introduces time relations in the mobility which we could not expect without(14). Hospital, commercial centers, uni-
versity etc.. are examples of chronotopoi. Even in presence of a chronotopic attraction the individual dynamics is not deterministic, but has a stochastic component with a finite correlation. Roughly speaking in this approach the urban mobility can be understood if one takes into account the presence of many citizens categories characterized by one or more chronotopic attractions (tendencies to mobility)(15), the internal dynamics of the chronotopoi that modulates the attraction and the urban space as a superposition of interacting communication networks associated with different type of transportation means and characterized by specific dynamical rules. As a consequence the urban mobility dynamical system has the features of a complex system, its evolution is probably far from an equilibrium state and the appearance of macroscopic laws can be related to the emergence phenomenon(4). In this paper we present a dynamical model that simulates this new approach to the problem of urban mobility and that shares some features with a Brownian agent model(16). The model describes the dynamics of the single individual by using a set of conditional probabilities that depend on the actual position in the urban networks, on the specific class the individual belongs to and on the past history. These probabilities make each individual to take a decision on the next step evolution. Each citizen should in principle obey to an internal scheduled program within a given time interval: i.e. to reach one or more chronotopoi and to spend some time in each one. The chronotopic attraction is given by a force depending on the distance from the chronotopos. No more than a single chronotopic force can be considered at each time step so that if
an individual feels the attraction of more chronotopoi, he/she makes a choice to define a time ordering of his/her tendencies. For the sake of simplicity we consider an urban space defined by two superimposed networks: the first corresponds to the pedestrian mobility and is defined by the roads network and the second corresponds to the public means mobility (user mobility) and it is defined by the public transportation lines. Each network is divided into nodes whose distance is covered in a unit time step. The networks communicate by special nodes (stations) where the citizen decides if it is convenient to take a train or not. The dynamics of the public means is purely deterministic and it is a priori given. Using a Manhattan-like roads network the simulations of urban mobility can be interpreted by using the mean field equations to compute the frequency of the three classes: users, waiting people and pedestrians. Due to the simplicity of the network geometry, it is possible to perform a continuum limit on biased random walk and to compute the users and pedestrians densities by solving two coupled Fokker-Planck equations when the chronotopoi are switched on. The comparison of the simulations with the analytical results allows to understand the influence of the microscopic dynamics and the boundary conditions on the global evolution of the model.

1 Description of the model

We will discuss the details of the urban mobility model in a case when two types of mobility are taken into account: the pedestrian mobility and the mo-
bility based on the public transportation. The main character of the model is the citizen that belongs to a class with a given request of mobility. The different categories and their relevance with respect the total population are suggested by the urban experts and the sociologists; examples of categories are businessmen, students, housewives etc.... The request of mobility is implemented by introducing an attraction towards the chronotopoi (see definition below). The citizens move in the urban space that is defined by the roads network. Each road is divided by equally spaced nodes and the pedestrian velocity corresponds to a jump between two nearby nodes in an elementary time step. At the crosses the choice among the different possibilities is made in a probabilistic way. Let $E_i$ be the event of choosing the $i$-th road at a given cross which connect $n$ roads (see fig. 1), the probability $P(E_i)$ is

$$P(E_i) = \frac{w_i p_i}{\sum_{j=1}^{n} w_j p_j}$$

(1)

where the quantity $w_i$ measures the attractiveness of the $i$-th road (i.e. the property of a road that could influence the citizen decision of choosing it) and $p_i$ is a probability computed by the citizen according to his request of mobility and his previous experience (memory effect). The attractiveness of a road is related to structural properties such as the width, the location inside a pedestrian area, the presence of shops or recreative centers, the presence of historical monuments or museums etc.... The probability $p_i$ depends on the individual tendencies, on the knowledge of the urban mobility state and on a memory
Fig. 1. Sketch of a cross node in the urban space; at each road is associated an attractivity weight $w_i$ whereas $p_i$ is the probability for a citizen to choose the $i$-th direction. $\vec{f}$ represents the chronotopic force at the cross.

effect and could be interpreted as a Bayesian-De Finetti probability(17). At the present computer implementation, if a citizen is incoming from the $j$-th road, the probability $p_i$ is computed according to

$$p_i = \begin{cases} 0 & \text{if } i = j \\ \frac{1}{n-1}(1 + f_i) & \text{if } i \neq j \end{cases}$$

(2)

where $f_i$ is the projection of the chronotopic force on the direction of the $i$-th road (see fig. 1). The definition (2) means that the pedestrian dynamics is a stochastic process with a deterministic drift force and a single time step memory; crowding effects can be inserted in the model as obstacles to the pedestrian mobility in the chosen direction(6; 20). A chronotopos is a special region of the urban space(13; 14) that influences the request of mobility of the citizens classes by means of a time dependent attractive force which modifies the probabilities $p_i$ of choosing a direction. The chronotopic force is a function of the Euclidean distance from the center of the chronotopic region and directed along the straight line that connects the considered node with the chronotopos center. However the direction is changed when the straight line
intersects a physical obstacle in the urban space as a river or the railway. In such a case we check if it is possible to overcome the obstacle (by using a bridge for example) within a certain tolerance in the direction otherwise we look for the closest direction to the original one with the requested property. Inside the chronotopic area the force is very low and the motion is almost a random walk, but people can be trapped for a certain time in special nodes (called topoi) that represent the activities inside the chronotopos itself. We distinguish two kinds of chronotopoi according to the different attractive laws: the strong chronotopoi like Hospital, University, working places whose attractive force is

\[ \vec{f} = c_1 r_c \hat{e}_c \]

and the weak chronotopoi like markets, shopping centers, cultural centers whose attractive force is

\[ \vec{f} = \begin{cases} 
  c_2 r_c \hat{e}_c & \text{if } r_c < R \\
  c_2 R^2 / r_c \hat{e}_c & \text{if } r_c > R 
\end{cases} \]

In eqs. (3) and (4) \( r_c \) is the distance and \( \hat{e}_c \) is the direction from the citizen position to the chronotopos center. \( R \) is a fixed distance and \( c_i \) are normalizing constants which define the ratio of the various chronotopic forces. The different types of mobility in the urban space are realized by introducing transportation networks superimposed to the roads network. In the present state of the model we have considered a network corresponding to the public trans-
portation (trains). The nodes of the network correspond to the stations and the velocity of the train is defined by the number of time steps required to go along the line between two nearby stations (this number is a property of the line). The trains move in the network in a deterministic way with a fixed distribution in the network and they reverse their direction at the headline stations. The citizens that have a tendency towards a chronotopos may decide to take a train with a probability $p_t$ proportional to the distance $r_c$ according to

$$p_t = \begin{cases} \frac{r_c}{r_0} & \text{if } r_c < r_0 \\ 1 & \text{otherwise} \end{cases}$$  \hspace{1cm} (5)$$

where $r_0$ represents the maximal walking distance covered by a pedestrian of a certain class. In such a case the direction $\hat{e}_c$ of the chronotopic force is moved in the direction of the nearest station. When the citizen reaches the station, he will be waiting until there arrives a train whose direction reduces the distance $r_c$ from the chronotopos. In such a case the citizen becomes a user and he will remain on the train as long as the train reaches the station at the minimal distance from the chronotopos. Then he decides to change train if there exists a line that reduces further on the distance from the chronotopos, otherwise he gets out of the station. The trains have a finite transportation capacity so that the waiting time depends both on the train frequency and on the number of users. We observe that the citizens do not have global information on the transportation network and there is no optimization in the choice of the train.
The urban mobility model has been implemented in a C++ code (MOBILIS)(18) to perform virtual experiments. For the sake of simplicity we have chosen a trivial geometry for the urban space (a Manhattan-like city) where the nodes of the pedestrian mobility network are uniformly distributed in the space and correspond to orthogonal crossroads. The space is homogeneous: i.e. all the roads are equivalent \( w_i = 1 \) cfr. eq. (1)) and at each node one has \( n = 4 \) possible orthogonal directions. At the boundary we have imposed absorbing condition for the citizens but the total number of citizens is constant; i.e. each time a citizen is absorbed another one is created randomly at one node of the network. The public means move at constant velocity along horizontal and vertical line (see fig. 2) and the stations are homogeneously distributed in the city every 5 crossroads so that the ratio between the train velocity and the pedestrian velocity is 5. The headline stations are at the border of the city and correspond to a reflecting boundary condition for the train dynamics. The train are randomly distributed along each line with a constant frequency. The choice of a trivial geometry for the simulation has two reasons: from one hand at the moment there are not enough experimental data on a realistic example to test the simulation results, from the other hand a simple city geometry allows a check of the virtual experiments by using a statistical approach (mean field theory). The core of MOBILIS is the updating of the pedestrian state according to the rules discussed in the previous section: at
each time step a citizen, which is not in a station, chooses in a probabilistic way the direction of the next step according to his chronotopic attraction; whereas a citizen at a station evaluates the convenience to take a train and eventually remains in the waiting state. The virtual experiments simulate the dynamics of the mobility in the urban space as a function of the time dependence of the chronotopic forces. The probabilistic character of the pedestrian dynamics associated with the chronotopic attractions mimics the zigzagging mobility that has been observed in the modern cities(11; 12) and may describe some experimental phenomena in a more realistic way than the OD models.

In fig. 3 we show 4 pictures of a virtual experiment with 2 strong chronotopoi
Fig. 3. Virtual experiment on a $25 \times 25$ grid for a Manhattan-like city with 10,000 citizens. The simulation considers the effect of 2 chronotopoi that switched on and off at different times. At each node the columns measures with different colors the density of pedestrians (yellow), users (blue) and waiting people (green). The pictures refers to different times of the simulations: in the top left picture the first chronotopos at the center of the urban space is switched on; in the top right picture we plot the stationary distribution at the first chronotopos; in the bottom left picture the first chronotopos is switched off and the second one in the upper right part of urban space is switched on; in the bottom right picture both the chronotopoi are switched off and the citizens distribution relaxes to a uniform distribution in the urban space.

(see eq. (3)) whose force field is switched on and off at different times. Three categories of citizens are considered in the simulation according to the different tendencies to mobility: the citizens that feel an attraction towards the first or the second chronotopos and the citizens that do not have any tendency. The total number of citizens in the simulation is 10,000 distributed on a grid of
25 × 25 nodes. At each node we plot a column with different colors that gives the density of the pedestrians, the users and the waiting citizens, so that the program allows an on-line monitoring of the urban mobility state during a simulation. The four pictures refer to different times: in the top left picture the first chronotopos in the center is switched on and one can see the citizens using the trains to reach the chronotopic area; in the top right picture the attracted citizens have reached the chronotopos and the presence of users is due to the other classes of citizens; in the bottom left picture the first chronotopos is switched off whereas the second one (in the upper right part of urban space) is switched on and the interested citizens move towards the second chronotopos; in the bottom right part the second chronotopos is switched off and the citizens are spreading in the urban space.

3 Mean field theory of urban mobility

In this section we consider an analytical approach based on average equations to describe the dynamics of the global variables $U(t), W(t)$ and $P(t)$, that define respectively the relative frequency of users, waiting citizens and pedestrians. We consider the Manhattan like geometry with a grid of $N_n = 101 × 101$ nodes and $N_s = 21 × 21$ stations uniformly distributed in the space; the number of citizens used in the simulation is 100,000. This choice allows to perform a continuum limit since the variation of the citizens distribution on a single time step or on the spatial scale between two nearby nodes can be neglected.
Table 1
Transition probabilities for the mean field equations

<table>
<thead>
<tr>
<th>Transition Probability</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{pu} )</td>
<td>( r - P_{pw} )</td>
</tr>
<tr>
<td>( P_{pw} )</td>
<td>( r(N_{hs}f^3_w + (N_s - N_{hs})f^4_w)/N_s )</td>
</tr>
<tr>
<td>( P_{up} )</td>
<td>( (1 - p_c)/\tau )</td>
</tr>
<tr>
<td>( P_{uw} )</td>
<td>( p_c((N_s - N_{hs})f^2_w + N_{hs})/(N_s\tau) )</td>
</tr>
<tr>
<td>( P_{wp} )</td>
<td>0</td>
</tr>
<tr>
<td>( P_{wu} )</td>
<td>1 - ( f^2_w )</td>
</tr>
</tbody>
</table>

The public transportation network contains \( 2N_s \) lines and the same number of trains \( N_t \) is randomly distributed along each line. The number of headline stations is \( N_{hs} = 84 \). The ratio between the train velocity and pedestrian velocity is \( v_{up} = 5 \). At first we consider the case without chronotopic attraction so that the space is homogeneous. The average dynamics of the global variables \( U(t), W(t) \) and \( P(t) \) satisfies the mean field equations

\[
\begin{align*}
P(t + 1) &= P(t) - (P_{pu} + P_{pw})P(t) + P_{up}U(t) + P_{wp}W(t) \\
U(t + 1) &= U(t) - (P_{up} + P_{uw})U(t) + P_{pu}P(t) + P_{wu}W(t) \\
W(t + 1) &= W(t) - (P_{wu} + P_{wp})W(t) + P_{uw}U(t) + P_{pw}P(t)
\end{align*}
\]

(6)

where the coefficients of the system (6) are the transition probabilities among the different states: i.e. \( P_{pu} \) is the transition probability from the pedestrian state to the user state and so on. The transition probabilities are computed as functions of the urban geometry according to the table 1 where \( r = N_s/N_n \) is the stations density, \( f_w = (N_s - N_t)/N_s \) gives the density of empty stations on a line, \( \tau \) is the average time spent by a user in a train (we have chosen \( \tau = 5 \) time steps corresponding to a journey of 5 stations) and \( p_c \) is the probability of changing the train direction (\( p_c = 0.5 \) in the simulations). In the definition of \( P_{pw} \) we have taken into account that the possible choice of a train is different for the internal stations (4 directions) and for the headline
stations (3 directions); $P_{uw}$ depends on the individual memory that prevents the choice of the opposite direction and on the fact that when a citizen changes train at a headline station he waits at least one time step. The equations (6) are no longer linear for all $t$ if the trains capacity is finite, because $P_{pu}$ and $P_{wu}$ are zero when the saturation threshold of chosen train is reached. The following conservation law holds: $P(t) + U(t) + W(t) = 1$ and two independent non-homogeneous difference equations can be written. The equilibrium solution $P_\infty$, $U_\infty$ and $W_\infty$ reads

$$
\begin{align*}
    P_\infty &= \frac{P_{up}P_{wu}}{R} \\
    U_\infty &= \frac{P_{wu}(P_{pu} + P_{pw})}{R} \\
    W_\infty &= 1 - P_\infty - U_\infty
\end{align*}
$$

(7)

where $R = P_{up}(P_{pw} + P_{wu}) + (P_{uw} + P_{wu})(P_{pu} + P_{pw})$. A generic solution of eq. (6) starts relaxing exponentially to the equilibrium solution and it is possible to study analytically the effect of an external forcing that describes the changing of the total number of citizens.

In fig. 4 we compare the solutions of eq. (6) with the simulation results for different values of $N_t = 5, 7, 10$ and infinite trains capacity; the chosen initial conditions are $P(0) = 1, U(0) = W(0) = 0$. The agreement is very good and shows the dependence of $W_\infty$ from $N_t$. In fig. 5 we plot the average mobility

$$
m(N_t) = P_\infty(N_t) + v_{up}U_\infty(Nt)
$$

(8)
Fig. 4. Comparison of the mean field solution $U(t)$ and $W(t)$ (continuous lines) with the simulation results (diamonds) without chronotopic attraction for different value of $N_t$ (the number of trains for each line): $N_t = 5$ left plot, $N_t = 7$ center plot, $N_t = 10$ right plot.

Fig. 5. Plot of the average mobility (see def. (8)) as a function of $N_t$ (the number of trains for each line).

as a function of the number of trains $N_t$. We see that $m$ increases rapidly at low values of $N_t$ and reaches the asymptotic value for $N_t \geq 10$ that corresponds to a trains density $\geq 1/2$ on each line.
The situation is more complicated when we consider the effect of a chronotopos. For $N_t = 0$ the citizens dynamics is a driven random walk on a uniform grid with a one step memory; the transition probabilities are given by eq. (2) with $n = 3$ and $f_i \ (i = 1, \ldots, 4)$ coincide with the projections of the chronotopic force on the coordinate axes. If the distribution of citizens $\rho_p(x, y, t)$ changes very little on the space scale given by the distance of two nearby nodes and on a time scale corresponding to a single integration step, then it is possible to perform a continuum limit and to write a Fokker-Planck equation

$$D_p^{-1} \frac{\partial}{\partial t} \rho_p = \left[ -\frac{\partial}{\partial x} f_x - \frac{\partial}{\partial y} f_y + \frac{1}{2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] \rho_p$$

(9)

where $D_p = \Delta x^2 / \Delta t$ is the diffusion coefficient. The effect of memory introduces a short term correlation in the pedestrian dynamics that can be neglected in the average evolution described by eq. (9). Moreover we remark that eq. (9) is separable so that the computation of $\rho_p$ reduces to the solution of a 1-dimensional equation. Since the chronotopic force admits a potential (cfr. eqs. (3) and (4)), there exists an asymptotically stable stationary solution of eq. (9) to which any solution converges with a characteristic time proportional to the second derivative of the potential at the chronotopos center (19). In the simulation we have scaled the urban space to the unit square $[-1, 1] \times [-1, 1]$ so that $\Delta x = .02$ since we have 101 nodes for each side. The diffusion coefficient is $D = 4 \times 10^{-4}$ (we set $\Delta t = 1$) and the chronotopos force is defined according to eq. (3) where $r_c \in [0, \sqrt{2}]$ and $c_1 = 1 / \sqrt{2}$ to satisfy the constrain
Fig. 6. Left plot: comparison of the chronotopos population computed by solving eq. (9) (continuous line) and by a simulation (diamonds) on a Manhattan like urban space with 100,000 citizens and without trains. After 300 time steps the chronotopos is switched off. Right picture: horizontal projection of the solution of eq. (9) at different times.

$p_i \geq 0$ in eq. (2). The chronotopos area is the square $[-.1,.1] \times [-.1,.1]$ and we have set the maximal walking distance (cfr. eq. (5)) $r_0 = .1$ that is the distance between 2 nearby stations. In the left figure 6 we compare the population in the chronotopos computed by solving the Fokker-Planck equation and by a direct simulation without trains ($N_t = 0$); the initial condition is a uniform pedestrian distribution in the urban space. The very good agreement proves that continuum limit is able to describe the evolution of average quantities. We observe that the relaxation time is $\simeq 100$ time step and that the stationary distribution is gaussian since the chronotopic potential is quadratic (see fig. 6 right). When we introduce the trains we have to consider the citizens dynamics on the public transportation network. Even if the stations distribution has coarse grained character, we still apply a continuum limit to describe the dynamics of averaged quantities.The user distribution $\rho_u(x,y,t)$ satisfies the
Table 2
Transition probabilities out of the chronotopos

<table>
<thead>
<tr>
<th>$P_{pu}$</th>
<th>$P_{pw}$</th>
<th>$P_{up}$</th>
<th>$P_{uw}$</th>
<th>$P_{wp}$</th>
<th>$P_{wu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r - P_{pw}$</td>
<td>$r(N_{hs}f_w + (N_s - N_{hs})f_w/N_s)$</td>
<td>$(1 - p_c)/\tau$</td>
<td>$p_c((N_s - N_{hs})f_w^2 + N_{hs})/(N_s\tau)$</td>
<td>0</td>
<td>$1 - f_w^2$</td>
</tr>
</tbody>
</table>

Table 3
Transition probabilities in the chronotopos

<table>
<thead>
<tr>
<th>$P_{pu}$</th>
<th>$P_{pw}$</th>
<th>$P_{up}$</th>
<th>$P_{uw}$</th>
<th>$P_{wp}$</th>
<th>$P_{wu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

same Fokker-Planck equation (9) with a diffusion coefficient

$$D_u = v_{up}^2 D_p$$

(10)

where the factor $v_{up}^2$ takes into account the train velocity. We couple the two distributions $\rho_p$ and $\rho_u$ by a local version of the mean field equations (6)

$$\dot{\rho}_p = -(P_{pu} + P_{pw})\rho_p + P_{up}\rho_u + P_{wp}\rho_w$$

$$\dot{\rho}_u = -(P_{up} + P_{uw})\rho_u + P_{pu}\rho_p + P_{uw}\rho_w$$

$$\dot{\rho}_w = -(P_{uw} + P_{wp})\rho_w + P_{wu}\rho_u + P_{pw}\rho_p$$

(11)

where $\rho_w(x, y, t)$ is the distribution of the waiting people and the transition probabilities depend on the positions. We distinguish two cases: if the nodes do not belong to the chronotopos the transition probabilities are computed according to the table 2 otherwise we use the table 3. In the table 2 we have chosen $\tau = 20$ to prevent that a citizen leaves a train before arriving at the chronotopos. The differences between the definitions in the tables 1 and 2 are due to the intelligent choice of the trains performed by the citizens (cfr. section 2) so that at a station (not at the border) only two directions satisfy
the condition of reducing the distance from the chronotopos. The values in the table 3 mean that a citizen leaves certainly a train when he arrives in the chronotopos and does not take any train in the chronotopos area. We expect that the complete system (two Fokker-Planck equation coupled by the system (11)) can describe the evolution of average variables if the train density on each line is not too small to avoid the effects of a granular distribution. In fig. 7 top part we compare the evolution of $P(t)$, $U(t)$ and $W(t)$ computed in the continuum limit and by a simulation in the case $N_t = 10$ with 100,000 citizens uniformly distributed at the beginning in the pedestrian state. The agreement between theory and simulation is very good: the initial increase of the users is due to the mean field equations (11) whereas the successive decreasing is the relaxation to the asymptotic solution of the Fokker-Planck equation with the user diffusion coefficients (10) when the users arrive to the chronotopos. In the bottom part of fig. 7 we compare the same quantities when $N_t = 1$; we remark that in this case the granular effect of the trains distribution prevents a good agreement of the simulation with the continuum limit approximation.

4 Conclusions

In order to study the urban mobility, especially the zigzagging one, we have built a dynamical system increasing step by step its complexity. At first the individual, equipped by a sort of genetic code which uniquely identifies it moves randomly on a streets network. At the second level the individual interacts
with a deterministic public transportation network and he/she can assume three dynamical states (p, pedestrian, u, user, w, waiting) with assigned transition probabilities. Finally we consider the urban activities (the urban life) modeled by the chronotopoi which act on citizens as attractive fields. An overcrowding parameter can model the cumulative effects of any cause tending to limit the individual mobility. In a physical language our system is quite similar to a statistical gas of elementary different particles which do not interact with each other and are subjected to several attractive fields. A check of the model with an active chronotopos is made by comparing the simulation results with the solution of two Fokker-Planck equations coupled by local mean field equations. To validate our system as a realistic one we need empirical tests on
the mobility in concrete cases and this is one of the next steps of our research program.

To conclude we point out that there are evident relationships between our model and other models of physical systems with increasing complexity like Brownian agent models. Biological systems, such as the immune one, have some similarities with our mobility network: in both cases there is a large number of elementary objects which move randomly being driven by some macroscopic fields. Thanks to its flexibility our model can be adapted to describe such systems (by introducing for instance two or more competing classes of elementary components) and may become a useful investigation tool in a multidisciplinary context.

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